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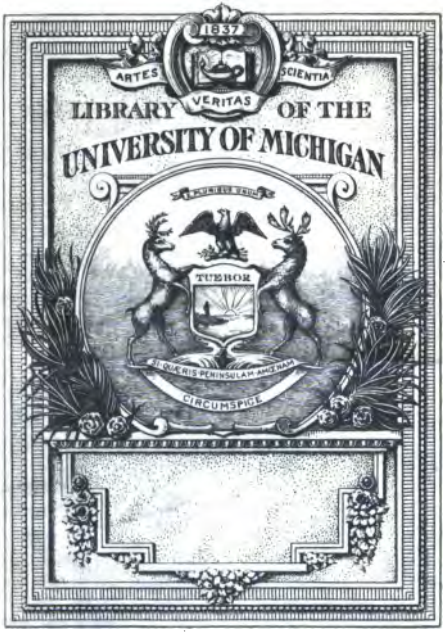


Stereometry,

Or, The ART of

GAUGING

Made eaſie, by the HELP of a

SLIDING-RULE:

Which ſhews the Area's of Circles in Gallons, and the Square and Cube-Root of any Number under 100000, by Inſpection: And is alſo very uſeful in *Arithmetick* and *Geometry*, particularly in the Meaſuring of *Superficies* and *Solids*.

WITH AN

APPENDIX

Of CONICK SECTIONS:

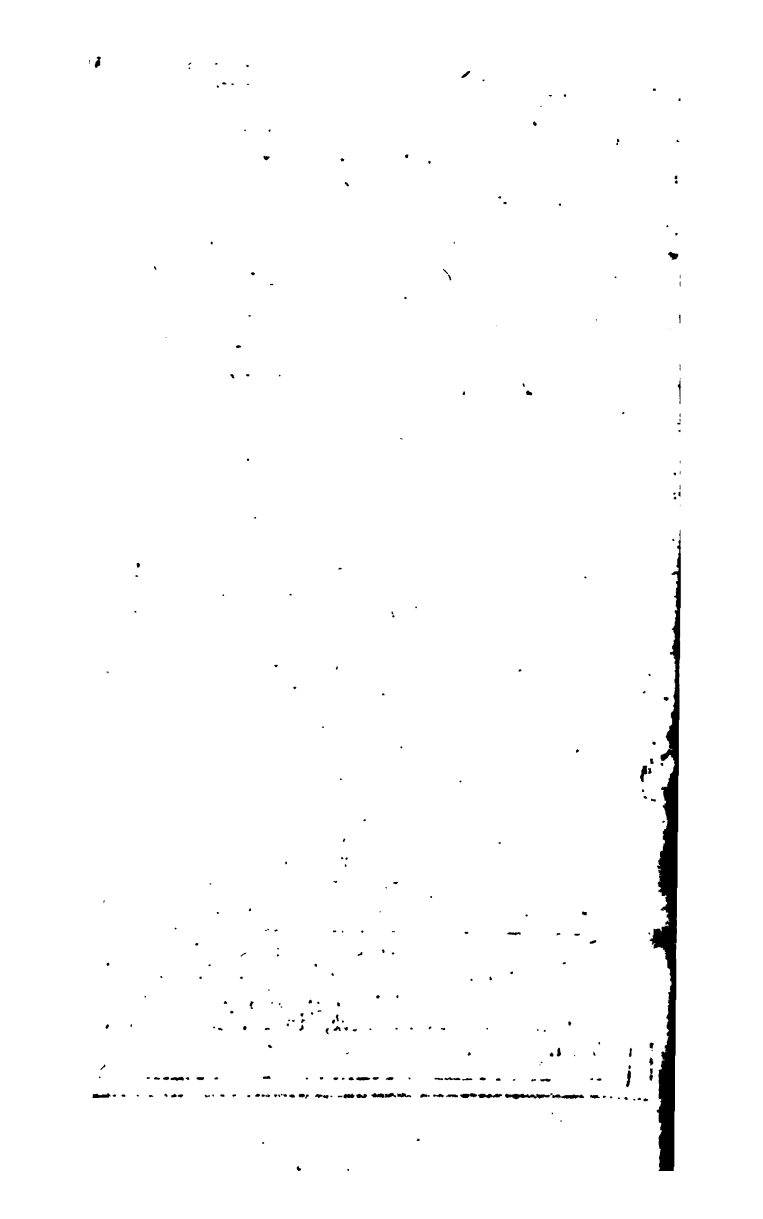
Shewing the *Nature*, *Properties*, and *Contents* of ſeveral *Solids*, frequently mentioned in Books of GAUGING.

The Eighth Edition, carefully Corrected.

To which are added ſeveral New TABLES, with Short and Plain Rules for Gauging of Malt.

By THO. EVERARD, Eſq;

LONDON:
Printed for *Mary Huſſey* in *Little-Britain*, *B. Tooke* at the *Temple-Gate*; *R. Wilkin*, *D. Midwinter*, *B. Cowſe*, and *G. Mortlock*, in St. *Paul's Church-Yard*. 1721.



TO THE

Right Honourable

SIDNEY

LORD *Godolphin*,

LATE

Lord HIGH-TREASURER

OF

ENGLAND;

This TRACT is Humbly

edicated and Presented

BY THE

AUTHOR.

Hist. 3 Sci.
Heffer
1-18-27
140497

THE PREFACE.

THE Inſtrument *deſcribed in this* Tract, (*tho' uſeful to all Perſons concern'd in the Meaſuring of* Superficies *and* Solids) *was principally deſign'd for the Aſſiſtance of the* Officers *of the* Exciſe; *for whom it is now found ſo uſeful and neceſſary, that no*

Gauger *is thought to be well prepared for his Buſineſs without it: But the many Thouſands that have been Sold, and the ſeveral Editions of this* Tract, *are ſo good Evidence of its Utility, that it is as needleſs, as unbecoming for me to ſay any thing in its Commendation.*

If the READER *finds this* Edition *ſomewhat different from ſome of the former, it is preſumed he will have no cauſe to complain of the* Alterations; *for beſides, that this is more* Portable, *and more* Correct, *here is alſo added* (Sect. IX.) *ſome Short and Plain*

Plain Rules for the Gauging of Malt; *with a particular Account of the* Dimenſions *and* Content *of the* Standard-Buſhel, *and how the ſame were found and aſcertained; alſo a* Table *ſhewing the* Diameter *and* Depth *of any legal* Buſhel, Half-Buſhel, Peck, *or* Gallon, *with* Directions *for making any of theſe* Meaſures, *and for Rectifying ſuch as are already made; with ſeveral other uſeful* Tables, *not hitherto made* Publick; *all which have render'd this* Tract *ſo compleat, that it is preſumed there will be no occaſion to* Alter *or* Add *to it in any future* Edition:

tion: *I have only this further to Advertiſe the* Reader *of*, viz.

That the Principal Lines upon this Inſtrument *are put upon a* Tobacco-Box, *to ſlide one by another, as they do upon the* Rule, *but with this conſiderable Advantage, that here the whole of one* Line *is always applied to the whole of another; ſo that all the Queſtions which may be reſolved by theſe* Lines *on the* Rule, *may be reſolved more readily by the ſaid* Lines *on this* Box, *and the* Directions *given in this* Tract *for the Uſe of the* Rule, *are alſo applicable to the*

the reſpective Lines upon the Box. *I ſhall Inſtance in one particular, which may ſerve inſtead of many; and that ſhall be in finding the* Area *of a* Circle *in* Ale-Gallons, *ſuppoſe the Diameter be* 40 *Inches.*

Set the Gage-Point *for* Ale-Gallons *upon* D, *to* 1 *upon the Line* C; *then againſt* 40 *upon* D, *you have* 4.45 *upon* C, *which is the* Area *required; and the* Lines *being thus ſet, you have in effect, a* Table *of the* Area's *of* Circles *in* Ale-Gallons, *to all* Diameters *whatſoever, without moving the Lines backward or for-*

forward, as muſt be done on the Rule *when one Line is drawn out beyond the End of the other.*

THE INTRODUCTION OF Decimal Arithmetick.

WHOEVER will rightly underſtand the Art of *Gauging*, or the *Uſe* of the *Inſtrument* here deſcribed, ought in ſome meaſure to be acquainted with the Art of *Arithmetick*; at leaſt with *Numeration*, *Addition*, *Subſtraction*, *Multiplication*, and *Diviſion*, both in *whole Numbers* and *Decimal Fractions*.

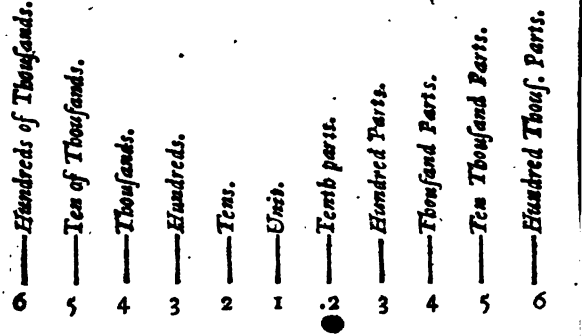
And foraſmuch as moſt of the *Problems* in this Tract are reſolved by help of the *Line of Numbers*, the ſeveral *Uſes* of which *Line* are render'd more eaſie by the knowledge of *Decimal Arithmetick*: I ſhall in the firſt place endeavour to ſhew what a *Decimal Fraction* is, and alſo give ſome Rules and Examples by which any Perſon who does at all underſtand the vulgar *Arithmetick*, may in a few hours time throughly comprehend this.

A *Decimal Fraction* is that, which by prefixing a Point or a Prick towards the left hand, its Value is decreas'd from ſo many Units, to ſo many tenth Parts of an Unit; and if a Point and a Cypher, or Digit be prefixed, it will then be ſo many hundred

parts, and if a Point and two Cyphers or Digits be prefixed, its value is decreased to be but so many thousand parts: As, if you would prefix before the Figure 2, a point thus [.2] 'tis then decreas'd from 2 Units, to 2 tenth parts of an Unit. For,

As in *whole Numbers* the Value or Denomination of places *increases* by Tens from the Unit's place towards the *left* hand; so in *Decimal Fractions*, the Value or Denomination of Places *decreases* by Tens from the Unit's place towards the *right* hand; as in the following Scheme.

In which I have set an Unit in its due place, and all the Figures towards the left hand (being whole Numbers) do *increase* by Tens; for the Figure 2, next the Unit's place is 2 Tens (or 20,) the next is 3 hundreds, and the next 4 thousands, &c.



But all the Figures from the Unit's place, towards the right hand (being *Decimal Fractions*) do *decrease* by Tens; so the Figure 2, towards the right hand is 2 tenth parts, the next is 3 hundred parts, and the next 4 thousand parts of an Unit; each place towards the right hand being ten times less.

In

In *Decimal Arithmetick*, we always imagine (and it would be very commodious if it were really so) that all intire Units, Integers, and Things, are divided into ten equal parts, and each of these parts so divided we call *Primes*; we also divide each of these *Primes* into other ten equal parts, and every of these Divisions we call *Seconds*; and each of these being divided into ten other equal parts, may be called *Thirds*, (and as one *Prime* is $\frac{1}{10}$ of an Unit, so one Second is $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{100}$ of an Unit, and one third is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{1000}$ part of an Unit) so by dividing the former, and subdividing these latter, we may run on *ad infinitum*.

Let a Pound Sterling be given to be Decimally divided:

According to the Notion premised, the first Division must be *Primes*, the next Division *Seconds*, the next *Thirds*, &c.

So if one Pound Sterling (being 20*s.*) be divided into ten equal parts, one of these parts is one *Prime*, and its value is 2*s.* and will stand thus [.1] three *Primes*, or three *tenths* of a Pound will stand thus [.3] and its value is 6*s.* Again, one *Prime*, or .1 being divided into ten equal parts, each of those parts will be one *Second*, or one hundredth part of a Pound, and is thus expres'd [.01] and its Value will be 2*d.* 1*f.* and $\frac{6}{10}$ of a Farthing; and if .01 be divided into ten other equal parts, each of those parts will be *Thirds*, or one *thousandth* part of a Pound, and will stand thus, [.001] and its Value will be $\frac{96}{100}$ of a Farthing.

The like may be understood of one Pound *Troy* or *Averdupois*, one Foot, one Gallon, or any other Integer, or Thing whatsoever.

A *Decimal* Fraction, whether it ſtand alone or be joined with *whole* Numbers, is diſtinguiſhed from a *whole* Number by a Point or Prick before it, towards the left hand, as in theſe Examples.

·4752 4·752 47·52 475·2

In *Decimal* Fractions the *Numerators* only are ſet down, the *Denominators* being known by the *Number* of Places in the *Numerator*; for if the *Numerator* conſiſt of but one Place, the *Denominator* is 10, if of 2 places, the *Denominator* is 100, *&c.*

Example.

The *Denominator* of { ·2, ·25, ·051, ·0752 } is { 10, 100, 1000, 10000 }

As Cyphers *before* a whole Number have no Value, ſo Cyphers *after a Decimal* Fraction are of no ſignification, and therefore will not increaſe the Fraction, nor alter the *Denominator*; for ·2 is two tenth parts, and ·20 is no more.

Again, as Cyphers *after* a whole Number do *increaſe* that Number, ſo Cyphers *before* a *Decimal* Fraction do *diminiſh* the Value thereof; as in theſe *Examples.*

·5, ·05, ·005, ·0005 } is 5 { *Tenth, Hundred, Thouſand, Ten Thouſand* } Parts.

Here

Here every Cypher added removes the Fraction further from Unity, making it ten times leſs than before.

Addition and Subſtraction of Decimal Fractions.

1. AS for the Operation of *Addition* and *Subſtraction* in *Decimals*, it is the *very ſame* with the *Vulgar*; there muſt only care be had of placing *Units* under *Units*, and *Fractions* under *Fractions* in their proper Ranks and Files.

For Example.

To	376.435
Add	54.22
Sum is	430.655

Again,

To	26.51
Add	527.342
Sum is	553.852

Subſtraction.

From	430.645
Subſtract	54.210
Remains	376.435

Or, thus:

From	553.852
Subſtract	527.342
Remains	26.510

Multiplication of Decimals.

1. IN *Multiplication* both of *Decimal* Fractions and *mixt* Numbers, there are ever *two* Numbers given to find a *third* unknown.

One of the Numbers given (and it's no matter which) is called the *Multiplicator*, the other the *Multiplicand*, and the Number ſought is called the *Product*; and this doth ever contain one of the Numbers given, as many times as the other given Number contains Unity.

Multiplication of *Decimals* is performed after the *ſame manner* as in *whole Numbers*; for the Numbers being ſet down one under another, we proceed in the Multiplication as if they were all whole Numbers; only it muſt be obſerved (when the Operation is finiſhed) how *many* places of *Decimal* Fractions are contain'd in the *Multiplicator* and *Multiplicand*, for *ſo many* muſt be in the *Product*, as in theſe Examples.

Mul-

Multiplicand ——— 35.25
Multiplicator ——— 7.24

14100
7050
24675

Product ——— 255.2100

Or, thus:

Multiplicand ——— 6.75
Multiplicator ——— .25

3375
1350

Product ——— 1.6875

2. When it happens (as it often does) that there are not so many *Figures* in the *Product*, as there are *Fractions* in the two Numbers to be multiply'd, you are then to place *Cyphers* before the *Product* till the *Number* of Places be equal; as in these Examples.

.04	.75	.85
.6	.05	.005
.024	.0375	.0425

3. As *whole Numbers* are increased by Multiplication, so *Decimal Fractions* are hereby made less; for the *Product* is removed farther from Unity than either of the Fractions given to be multiplied; as appears by the last Examples.

4. When a *Decimal* Fraction or *mixt* Number is to be multiplied by an Unit with Cyphers (as 10, 100, 1000, &c.) you need only remove the Point ſo many places towards the right Hand, as there are Cyphers with the Unit; thus, if .7562 be multiply'd

By	The Product will be
10	7.562
100	75.62
1000	756.2
10000	.7562

Diviſion of Decimals.

1. IN *Diviſion* there are *two* Numbers given, to find a *third* unknown.

One of the Numbers given is called the *Diviſor*, the other the *Dividend*, and the Number ſought is called the *Quotient*, and this *Quotient* doth ever contain Unity as many times as the *Diviſor* is contained in the *Dividend*.

Diviſion of *Decimal* Fractions or *mixt* Numbers, is performed after the *ſame manner* as in *whole* Numbers; all the Difficulty is to know the Denomination of places in the *Quotient*, and they are eaſily diſcovered by this one general Rule.

The firſt Figure in the Quotient muſt be of the ſame Denomination with that Figure in the Dividend, which at the firſt demand ſtands exactly over the place of Unity in the Divi-

Diviſor. I ſhall make this plain by ſome Examples.

Example.

Let it be required to divide 32 *by* 51.2.

In this Example the *Diviſor* (51.2) is *greater* than (32) the *Dividend*, therefore in this and the like caſes you muſt place a competent Number of *Cyphers* behind the *Dividend*, which (if it be a whole Number) the Cyphers ſo placed muſt be diſtinguiſhed there-from as Fractions; as here I put 4 Cyphers behind 32, and placing the Numbers in this manner,

51.2) 32.0000 (

I proceed to the Diviſion (as if all were whole Numbers) and firſt ask, how many times the *Diviſor* is contained in (320) the three firſt Figures of the *Dividend?* And ſeeing in this Example it is not contained in the three firſt Figures, it will therefore extend to the fourth place, over which I put a prick, to ſhew that I muſt begin to ſubſtract at that place. I then ask how many times 5 in 32? I find 6, I place 6 in the *Quotient*, and multiply the *Diviſor* by 6, Subſtracting the Product out of the *Dividend*, beginning my Subſtraction at the place over which I put the prick: Thus 6 times 2 is 12, 12 from 0 I cannot, I borrow 2, and ſay 12 from 20, and there remains 8, which I place under the Cypher: Again, 6 times 1 is 6, and 2 I borrowed is 8; 8 from 10, and there

51.2) 32.0000 (6
128

there reſts 2, which I place under the next Cypher before 8, as in the Example; then 6 times 5 is 30, and 1 I borrowed is 31; 31 from 32, and there reſts 1, which I ſet under 2; then for my new Dividend I bring down the next Cypher, and ſet it behind the Remainder, and the Example will ſtand thus:

51.2) 32.0000 (6
1280

Then proceeding in my Diviſion, I ask how many times 5 in 12, I find 2, which I place in the *Quotient*, and ſay 2 times 2 is 4; 4 from 10, and there remains 6, which I ſet under 0: Again, 2 times 1 is 2, and 1 I borrowed is 3; 3 from 8, and there reſts 5, which I place under 8; then 2 times 5 is 10; 10 from 12, and there reſts 2, which I place under 2, and to this Remainder I bring down another Cypher to make my new *Dividend*; ſo will the Example ſtand thus:

51.2) 32.0000 (62
1280
256

51.2) 32.0000 (62
1280
2560

This done, I enquire how many times 5 in 25, and find 5, which I place in the *Quotient*, and ſay 5 times 2 is 10, 10 from 10 and there reſts 00; then 5 times 1 is 5, and 1 I borrowed is 6, 6 from 6, and there reſts 00. Laſtly, 5 times 5 is 25, 25 from 25 and there remains 0; and thus my

my Diviſion is finiſhed, and the Example will ſtand thus:

51.2) 32.0000 (.625
1280
2560
0000

Now by the general Rule above given, it is plain that the firſt Figure in the *Quotient* muſt be Tenths, and therefore the Quotient is all Decimal parts; for the *Diviſor* being placed under the firſt Figures of the *Dividend*, this Example will ſtand thus:

32.0000 (.625
— 51.2

By which it appears, that the *Cypher* in the *Dividend*, which ſtands over the *Unit's place* in the *Diviſor*, is the place of *Tenths*.

See more Examples.

Diviſor.	*Dividend*	*Quotient.*
51.2)	320,000	(6.25
52.2)	32.0000	(.0625
7.5)	.375	(.05

2. In Diviſion of *whole* Numbers or *mixt*, if there happen to be a Remainder, you may bring down more Cyphers, and by continuing your Diviſion, carry on the *Quotient* to as many places of Fractions as you pleaſe; four or five will be very near the truth, but two or three will be ſufficient in moſt Caſes.

Example;

Example.

6.26) 25.800000 (4.1214
760
1340
880
2540
36

3. When any Number (either *Decimal* or *mixt*) is to be divided by an Unit with Cyphers, as 10, 100, 1000, *&c.* you need only remove the prick in the *Dividend* ſo *many* places toward the left Hand, as there are *Cyphers* with the Unit, ſupplying the *vacant* places (if any be) with Cyphers.

Thus 756.2 divided by 10, is 75.62, divided by 100, it will be 7.562, by 1000, .7562, and if by 10000, the *Quotient* will be .07562.

4. *Diviſion* is proved by *Multiplication*; and *Multiplication* by *Diviſion.*

To *prove Diviſion*, multiply the *Quotient* by the *Diviſor*, and the *Product* is the *Dividend:* Thus, .652 (the *Quotient* in the firſt Example of *Diviſion*) being multiplied by 51.2 the (*Diviſor,*) the Product is 32.0000 (the *Dividend.*)

To *prove Multiplication*; divide the *Product* by either of the Numbers given to be multiplied, and the *Quotient* will be the other Number given.

Thus if the laſt *Product* (*viz.*) 32.0000 be divided by 51.2, the *Quotient* is .625, or of 32.000 be divided by .625 the *Quotient* will be 51.2.

And hence 'tis evident, that *Diviſion* is alſo *proved* by *Diviſion*: For if any *Quotients* be made a *Diviſor*, it will quote the firſt *Diviſor.* Example:

1.8)

1.8) 28.8 (16
108
000

16) 28.8 (1.8
128
000

5. Now foraſmuch as *Multiplication* is *eaſier* than *Diviſion*, I ſhall here ſhew, that what is performed by a *Diviſor*, may alſo be performed by a *Multiplicator*, and how ſuch *Multiplicator* may be found: Suppoſe I were to *divide* 900 by 25, the *Quotient* will be 36.

Now let it be required to *multiply* 900 by a certain Number that ſhall produce 36.

To *find* this *Multiplicator*, *divide* an Unit with Cyphers by 25, the *Diviſor* propoſed, the *Quotient* will be .04 the *Multiplicator* ſought; for 900 being *multiply'd* by .04, the *Product* will be 36, which was required.

Again, Having a *Multiplicator*, to find a *Diviſor*, this is but the *Converſe* of the former; for if an Unit with Cyphers be *divided* by .04 (a *Multiplicator*) the *Quotient* will be 25, the *Diviſor* as before.

6. A *Vulgar* Fraction is reduced into a *Decimal* by *Diviſion*: For which the Rule is,

Divide the *Numerator* (of the *Vulgar* Fraction given) by the *Denominator*, and the *Quotient* will be a *Decimal* Fraction, *equal* in Value to the *Vulgar* Fraction given: So $\frac{3}{4}$ reduced as aforeſaid will be .75: *Example.*

4) 3.00 (.75
20
00

Note,

Note, The *odd Parts* of an Integer cannot be exactly reduc'd to a *Decimal*, for there will always be a *Remainder*; in such Cases carry on the Fraction to four or five places, it will be very near the Truth. *Example*;

$\frac{4}{9}$ 9) 4,0000 (.4444
40
40
40

7. To reduce a *Decimal Fraction* to the *known parts* of the *Integer*:

If the *Decimal* given be part of a *Pound* Sterling, multiply it by 20 (the *Shillings* in a Pound) and the Fractions in the Product by 12 (the Pence in a Shilling) and the Fractions in this Product by 4, (the *Farthings* in a Penny.) Then the *whole* Numbers in the respective Products shew the *Shillings*, *Pence* and *Farthings* contained in the Decimal given, and the *Fractions* in the last Product are Decimals of a Farthing. *Example*; Suppose .25 a Decimal Fraction of a Pound Sterling, were given to be reduc'd as aforesaid: Multiply it by 20, the Number of Shillings in a Pound, the Product will be 5, *viz.* 5 *s.*

.25
20
Shillings——5,00

So the *Value* of .60625 will be found to be 12 *s.* 1½ *d.* As appears by the following Example:

.60625

.60625
20

Shillings ——— 12.12500
12

25000
12500

Pence ——— 1.50000
4

Farthings ——— 2.00000

If the Decimal given be part of a *Beer Barrel*; multiply it by 4, (the *Firkins* in a Barrel) and the Fractions in the Product by 9 (the *Gallons* in a Firkin,) and the Fractions in that Product by 8, (the *Pints* in a Gallon.) The *whole* Numbers in the respective Products are equal to the *Firkins*, *Gallons* and *Pints* contained in the Decimal propounded.

Example.

Let .72 be a Decimal of a Beer Barrel to be reduced as aforesaid, it will be found to be 2 Firkins 7 Gallons, 7 Pints, and .36 of a Pint, as appears by

.72
4

Firkins ——— 2.88
9

Gallons ——— 7.92
8

Pints ——— 7.36

But

But *theſe* and *many other* Arithmetical Queſtions may be more *readily reſolved* by the *Inſtrument*; a Deſcription of which ſee in the firſt Section of the following Diſcourſe.

STE-

STEREOMETRY,

OR, THE

ART of GAUGING

MADE EASIE

By the HELP of a New

SLIDING-RULE.

SECT. I.

A Deſcription of the Rule, and of the ſeveral Lines upon it, with their Uſe in ſome Queſtions in Arithmetick.

THE Rule conſiſts of Three Parts, *viz.* a Rule of 12 Inches long, and two ſmall Scales to ſlide in it, which may be drawn out, one towards the right Hand, and the other towards the left, till the whole be three Foot long.

The principal Lines on the Inſtrument are thoſe commonly known by the Name of *Gunter's-Line*, or

or *Line of Numbers*, which are here diſtinguiſhed one from another by certain Letters, ſet at the end of the Lines towards the right hand.

Thus the Lines D, are each of them one *ſingle Line of Numbers*, beginning at the End of the Rule towards the left hand, and from thence continued to the other End.

The Lines AB, and C, are called *double Numbers*, each being *two* Lines or Radius's of Numbers, the *firſt* beginning at the *left* hand, and ending in the *middle* of the Rule *where* the *ſecond* Radius begins, and is from thence continued to the *end* at the *right* hand.

The Line E is call'd *Triple Numbers*, being *three* Radius's of Numbers; the *firſt* beginning at the *left* hand, and the *third* ending at the *right* hand.

This *Triple* Line is *equal* in length to the *double* Lines, and all to the *ſingle* Line, for all the *five* begin and end at the *ſame* Point.

The *Line of Numbers* is (now) ſo *well known* to moſt Perſons, that it may be thought a *ſufficient* Deſcription to have only ſaid that *theſe* are *ſuch*.

But for as much as this Inſtrument may be uſeful to ſome who (I preſume) do not yet know what the *Line of Numbers* is, I ſhall therefore endeavour to explain it as followeth.

The *Line of Numbers*, is a Line of Geometrical Proportions, divided firſt into *Nine unequal* Parts, call'd *Primes*, which are diſtinguiſh'd by Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; and then each of theſe *Primes* are ſubdivided into *ten* other parts (according to the ſame Reaſon) call'd *Tenths*, and again each of thoſe *Tenths* are ſubdivided, or at leaſt ſuppos'd to be ſubdivided into *ten* other parts, according as the *length* of the Line will admit; as here the *Line* D, being about 11 Inches long, each tenth in the firſt *Prime* is

is really ſubdivided into *ten* parts, call'd *Centeſms*, but betwixt the Figures 2 and 4, each *tenth* is divided but into five parts, and therefore each of thoſe parts do ſignifie *two Centeſms:* Again, from the Figure 4 to the end of the Line, every tenth is divided into two parts, each repreſenting five *Centeſms:* Laſtly, each of theſe *Centeſms* is alſo ſuppoſed to be divided into *ten* parts, which by ſome are called *Millions*; but a Line of this Length will not admit of this laſt diviſion.

The Figures (1, 2, 3, 4, 5, 6, &c.) by which the *Primes* are diſtinguiſh'd, are all arbitrary Points, and may any of them repreſent ſo many entire *Units*, *Tens*, *Hundreds* or *Thouſands*; or they may alſo repreſent ſo many *Tenth*, *Hundredth*, *Thouſandth*, or *Ten Thouſandth* parts of an *Unit*.

1. For *whole*, or *intire Units*; Let the firſt *Prime* or the Fig. 1. at the beginning of the Line D, repreſent one *Unit*, then ſhall all the Figures towards the right hand, (*viz.* 2, 3, 4, 5, &c. to 10.) repreſent *ſo many* Units, and the tenths in each *Prime*, will be *tenth parts*, and the *Centeſms* in each of thoſe tenths will be *hundredth parts* of an Unit.

Or, let 1 at the beginning of the Line repreſent 10 Units, then will each *Prime* forwards repreſent 10 times ſo many Units as the Figures expreſs; thus, the Figures 2, 3, 4, 5, &c. will be 20, 30, 40, 50, &c.

And when one *Prime* repreſents 10 *Units*, every *tenth* in that *Prime* will be 1 *Unit*, and each *Centeſm* in thoſe *tenths* will be 1 tenth part of an *Unit*.

Again, Let the firſt *Prime* repreſent 100, then the Figures 2, 3, 4, 5, &c. will repreſent 200, 300, 400, 500, &c. and therefore 10 at the end will be 1000; and according to this Suppoſition, 1 tenth in each *Prime* will be 10 Units, and in thoſe tenths each *Centeſm* will be 1 Unit.

2. For

2. For *Decimal Fractions*; Let 10 at the end of the Line (at D) repreſent 1, then each *Prime* towards the left hand will be .1, and in thoſe *Primes* each tenth will be .01, and in theſe tenths each *Centeſm* will be .001 part of an Unit.

To make this more plain, draw out the ſliding Piece B, till 1 at the *beginning* of the Line B, ſtand exactly againſt 10 at the *end* of the Line A, for then you have a Line of Numbers four times repeated; upon which, let 1 at the beginning of the Line A, repreſent 1 Unit; then ſhall 1 in the *middle* of the ſaid Line be 10; and 10 at the *end* thereof (or which is all one, at the *beginning* of the Line B,) repreſent 100; and by conſequence 1 in the *middle* of the Line B will be 1000, and for the *ſame* reaſon 10 at the end of the ſaid Line (which is alſo the *end* of the *Rod*) will be 10000.

But keeping the Rule as it now ſtands, let 10 at the end of the fourth Radius (*viz.* at B.) repreſent 1, then ſhall each Prime in the ſaid fourth Radius repreſent .1, in the third Radius .01, in the ſecond Radius .001, and in the firſt .0001 part of an Unit.

So will 2 in the firſt Radius be .0002, in the ſecond .002, in the third .02, and in the fourth .2 parts.

The *Numbers* and *Diviſions* on the Lines being thus explain'd, it will not be difficult to find the Point upon the Line, where any Number given is repreſented.

As for Example; Suppoſe the Numbers were 1895, for the firſt Figure thereof (*viz.* 1.) I count 1 at the beginning of the Line D, for the ſecond Figure I count 8 tenths next following (that is 8 of the greater Diviſions betwixt 1 and 2.) then from this Point forward I count 9 *Centeſms* for the third Figure, and

for

for the laſt Figure 5, I count half the next *Centeſm*; ſo I find the Point *a g*, will repreſent 1895, and by the ſame Rule the Number 1715 will be found at the Point *w g*; hence obſerve,

1. That on a Line of this length, only the *four firſt* Figures of any Number propos'd can be diſcover'd, for if the Number given were 189582, it would be repreſented at the ſame Point where the former 1895 was found (*viz.*) at *a g*.

2. That all Numbers which after the firſt Figure have nothing but *Cyphers* (as 20, 200, 2000, *&c.*) are all repreſented at the *ſame* Point.

So the Numbers 20, 200, 2000, are all repreſented by the Figure 2, at the beginning of the ſecond *Prime*.

3. All Numbers conſiſting of *three* Figures, and having a Cypher in the *middle*, are found within the firſt tenth of that *Prime*, at which the firſt Figure of the Number given is found.

Example, Let the Number given be 308, for the *firſt* Figure I count 3 on the Line (which I find at the beginning of the third *Prime*) now there being a *Cypher* in the *ſecond* place, I muſt not count any of the tenths, but for the *laſt* Figure 8, I count 8 *Centeſms*, and that is the Point which repreſents 308.

4. All Numbers conſiſting of *four* places, and having *two* Cyphers in the *middle*, muſt be ſought betwixt the beginning of the *Prime* unto which they belong, and the firſt *Centeſm* of the ſame *Prime*; ſo 4005 being given, the *firſt* Figure (*viz.*) 4, is found at the beginning of the fourth *Prime*: Now there being Cyphers in the ſecond and third places, I muſt not count any of the tenths or *Centeſms*; but for the *laſt* Figure 5, I eſtimate 5 *Millions*, which is about the middle of the firſt *Centeſm*, and that is the Point where 4005 is repreſented.

Note,

Note, *Decimal Fractions* and *mixt Numbers* are discover'd after the *same* manner as whole Numbers; for if the first Number 1895, were 18, 95, it will be found at the *same Point, (viz.)* at *a g*; and by what hath been said, it is ease to find what Number is represented at any Point upon the Line, as will appear in the following Questions of Proportion, of which I come now to treat.

In *Arithmetick* I shall take notice of two sorts of Proportion, *Arithmetical* and *Geometrical*.

1. *Arithmetical* Proportion is when *divers* Numbers, being *compared* together, retain amongst themselves equal Differences, as these, 2, 4, 6, 8, *&c.* And this is either *continued*, as in the Numbers before propounded (which is also called *Arithmetical Progression*, or a *Rank* of Numbers *Arithmetically* proportional;) or *discontinued*, as in these, 2, 4, 8, 10, or the like.

2. *Geometrical* Proportion, is when *divers* Numbers being compared together, *differ* amongst themselves, according to the same Rate or Reason, as these 2, 4, 8, 16, *&c.* for here, as 2 is half 4, so 4 is half 8, and 8 half 16; this is likewise either *continued*, as in those before propounded (which are also called *Geometrical Progression*, or a *Rank* of Numbers *Geometrically* proportional;) or *discontinued*, as in these, 2, 4, 16, 32; for as 4 is double 2, so is 32 double 16, but so is not 16, being compar'd with 4.

Pro-

PROBLEM I.

Having two Numbers given, to find a third Geometrically proportional unto them, and to three a fourth, and to four a fifth, &c.

FInd *one* of the Numbers given upon the Line B, and ſet it againſt the *other* given Number on the Line A, then find the *ſame* Number upon B (which was laſt counted upon A) and againſt it you have this *third* proportional upon A; and againſt this *third* upon B, is the *fourth* upon A; in like manner, againſt the *fourth* upon B, you have the *fifth* on A, *&c.*

Example, Let it be required to find a *third* proportional to *theſe two* Numbers 2 and 4, which may bear the *ſame* proportion to 4, that 4 bears to 2.

Draw out the ſliding Rod till 2 upon B ſtand againſt 4 upon A, then againſt 4 upon B is 8, (the *third* proportional) upon A; and againſt this *third* (*viz.* 8) upon B, is 16 upon A; which is the *fourth* proportional: Likewiſe againſt 16 upon B, is 32 the *fifth* upon A, and againſt 32 upon B, is 64 the *ſixth* proportional: But now proceeding forward, I find that 64 upon B, will reach beyond the end of the Line A, I therefore ſeek 64 towards the *left* hand upon B, and againſt it I find 128 the *ſeventh* proportional; and ſo proceeding further, you may find the *eighth* to be 256, the *ninth* 512, *&c.* *Contrariwiſe,* if it were required to find a *third* proportional to the *ſame* Numbers 2 and 4, which may bear the *ſame* proportion to 2, that 2 bears to 4.

Set

Set 4 in the ſecond Radius upon A, to 2 upon B, then againſt 2 upon A (towards the left hand) is 1, the *third* proportional, and againſt 1 upon A is .5 the *fourth* upon B, alſo againſt this *fourth* (*viz*) upon A is .25, the *fifth* proportional on B, *&c.*

Multiplication by the Lines.

Problem IJ.

One Number being given to be multiplied by another, to find the Product.

IN Multiplication either of whole Numbers, mixt or decimal Fractions; the Proportion is:

As 1 *is to the Multiplicator;*
So is the Multiplicand to the Product.

And the *Product* of any *two* Numbers ſhall have ſo *many* places as there be in *both* the Numbers given, except when the *leſſer* of them do not exceed ſo *many* of the *firſt* Figures of the Product; then it will have *one* leſs.

1. Example: *Let it be required to multiply* 6 *by* 4.

Say, [1 : 4 : : 6 : 24] which Analogy or Proportion may be read thus; *As* 1 is to 4, *So is* 6 *to* 24.

Therefore, Set 1 upon the Line B, to 4 on the Line A, and then againſt 6 upon B, is 24 upon A; which is the Product ſought.

Note, The *Unit* or *firſt* term may be taken upon *either* of the Lines A or B, but the *firſt* and *third* terms muſt be counted upon *one* and the *ſame* Line, and

and the *Second* on the *other* Line, where the *fourth* will alſo be found.

The Letters A and B may ſerve to diſtinguiſh the Lines.

2. Example: *Let the two Numbers given be* 68 *and* 26, *to find the Product.* The Proportion is, [1 : 26 : : 68 : 1768.]

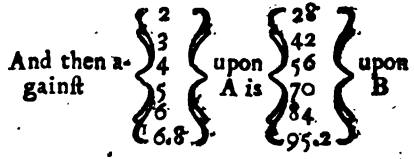
Therefore, Set 1 upon B, to 26 upon A, then againſt 68 upon B, is 1768 on A, which is the Product ſought; Or,

Set 1 upon B, to 68 upon A, then againſt 26 upon B, is 1768 (on A) the Product. Therefore it matters not *which* of the Numbers given be made the *Multiplicator*; and *note* alſo, that the Product hath as many places as are in both the Numbers given, becauſe the leaſt of them (*viz.* 26) *doth* exceed ſo many of the firſt Figures of the Product, according to the Rule before given.

3. Example: *Let* 68 *be multiply'd by* 14. The Proportion is, 1 : 14 : : 68 : 952.

Therefore, Set 1 upon B, to 14 upon A, then againſt 68 upon B, is 952 the Product upon A, and here the Product conſiſts of one place *leſs* than there be in the two Numbers given, becauſe the leſſer of them (*viz.* 14) doth *not* exceed ſo many of the firſt Figures of the Product.

Now that the Product laſt found is 952, and not 95.2, nor 9.52, will thus appear: Set 1 upon B, againſt 14 upon A.



By this 'tis evident, that the Lines are in effect a Table of Multiplication, for having set 1 to the Multiplicator, against any Multiplicand, you have the Product; so if 2 be multiply'd by 14, the Product is 28; if 3, the Product will be 42; if 4, the Product will be 56; if 5, 70, &c. Hence I conclude, that if the Multiplicand had been but 6.8, the Product would have been 95.2, but the Multiplicand being 68, the Product must be 952; for by taking away the prick, the Fractions in each are made whole Numbers.

When of *two* Numbers given to be multiply'd, the *one* consists of whole Numbers or *mixt*, and the *other* of Fractions only, make the *whole* or *mixt* number the *Multiplicator*, and having set 1 against it, seek the Fraction towards the left hand, and against it you have the *Product*.

4. Example: *Let the two Numbers be* 27.5, *and* .8.

Set 1 on A, to 27.5 on B, and then against .8 (which being *less* than 1, I seek towards the left hand) on A, is 22 on the Product on B.

And notwithstanding a Number of *more* than four places cannot be exactly expressed on a Line of this length, yet the Product of any Multiplication may be discover'd to *six* or *seven* places at least.

5. Ex-

5. Example: *Suppoſe I were to multiply* 2482 *by* 54. The proportion is,

1 : 54 : : 2482 : 134028

Therefore ſet 1 upon A to 54 upon B, and then againſt 2, upon A, is 108 upon B: Now ſuppoſe,

$$2 \text{ be } \left\{\begin{matrix}20\\200\\2000\end{matrix}\right\} \text{ 108 will be } \left\{\begin{matrix}1080\\10800\\108000\end{matrix}\right\}$$

So the Product will have ſix places, and againſt 2482 the Multiplicand, you may diſcern (upon the Line) the four firſt of them, (*viz.*) 1340; the two laſt may be found by multiplying (in ones mind) the two laſt Figures of the *Multiplicand*, by the two laſt of the *Multiplicator*; for ſo ſhall you diſcover the two laſt Figures of the *Product*, which in this Example will be 28, which plac'd behind the four firſt (*viz.*) 1340, makes the Product compleat 134028.

I might here add more Examples, but theſe already given may ſerve, they containing (I think) ſufficient Directions for all the Variety that can happen in Multiplication.

Diviſion by the Lines.

PROBLEM III.

One Number being given to be divided by another, to find the Quotient.

IN Diviſion both of whole Numbers and mixt, the Proportion is,

As the Diviſor is to 1,
So is the Dividend to the Quotient.

Which Quotient ſhall ever conſiſt of ſo many Figures as the Dividend hath more than the Diviſor, except when the Diviſor does *not* exceed ſo many of the firſt Figures of the Dividend; then it ſhall have one place more.

1. Example: *Let it be required to divide* 24 *by* 4; The Proportion is, 4 : 1 : : 24 : 6.

Therefore, Set 4 upon B, to 1 upon A, and then againſt 24 upon B, is 6 upon A, which is the Quotient ſought.

2. Example: *Let it be required to divide* 1768 *by* 26: The proportion is, 26 : 1 : : 1768 : 68.

Therefore, Set 26 upon B, to 1 upon A, and then againſt 1768 upon B, is 68 (the Quotient) upon A.

3. Example; *Suppoſe* 952 *were to be divided by* 14: The Proportion is, 14 : 1 : : 95 : 68.

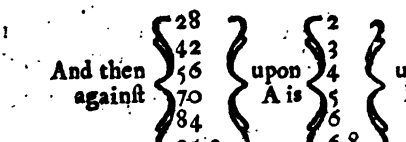
There-

Therefore, Set 14 upon A, to 1 upon B, then against 952 upon A, you have 68 upon B, which is the Quotient requir'd.

Observe here, the Dividend hath but one Figure more than the Divisor, yet the Quotient consists of two Figures, because the Divisor doth* *not* exceed so many of the first Figures of the Dividend; but in the First and Second Examples the Quotients have but so many Figures as the Dividend hath more than the Divisor, because the Divisor doth *not* exceed so many of the first Figures of the Dividend, according to the general Rule above given.

This shews (in all Cases) how many Figures must be in the Quotient, the Value of the first of which may be found by the Rule given in the Introduction, *Pag.* 8.

Or it will thus appear, that the Quotient in the last Example is 68, and not 6.8, nor .68: Set 14 upon A, to 1 upon B,



This is but the Converse of the Third Example of Multiplication; and as by that it is manifest, that if 2 were multiplied by 14, the Product would be 28; if 3, it would be 42; if 4, 56, *&c.* So here it is evident, that if 28 be divided by 14, the Quotient will be 2; if 42, it will be 3; if 56, it is 4, *&c.* thus by reading on the Proportion from the

Diviſor 14: 28, 56, 70, &c. I find at laſt, that if the Dividend were but 95.2, the Quotient would be 6.8, but the Dividend being 952, the Quotient muſt be 68; for by taking away the prick, the Fractions in each are made whole Numbers.

By theſe Examples it is alſo evident, that at *once* ſetting of the Rule we both *Multiply* and *Divide*.

For if 14 be a Multiplicator, ſet 1 on B, againſt 14 on A; then againſt any Multiplicand upon B, you have the Product upon A; as appear'd by the third Example of Multiplication.

And without moving the Rule, if you ſuppoſe 14 to be a Diviſor, then againſt any Dividend upon A, you have the Quotient upon B; as in the laſt Example of Diviſion.

How by *any Diviſor* to find a *Multiplicator*, was ſhewn in pag. 13 of the *Introduction*; it may alſo be perform'd by the *Lines* more readily.

Thus *having* a Diviſor, to find a Multiplicator;

Set the Diviſor given upon A, to 1 upon B, and then againſt 1 (towards the left hand) upon A, is the Multiplicator upon B. Example; Suppoſe 25 were a *Diviſor* given to find a *Multiplicator*.

Set 25 upon A, to 1 upon B, and then againſt 1 (towards the left hand) upon A, is .04 the *Multiplicator* ſought.

By a *Multiplicator* to find a *Diviſor*: This is but the Converſe of the former; for having .04 a Multiplicator upon B, ſet it againſt 1 upon A, and then againſt 1 upon B, is 25 the Diviſor, as before.

Pro-

Problem IV.

To reduce a Vulgar Fraction into a Decimal.

SET the Denominator (of the Vulgar Fraction given) upon A, to the Numerator thereof upon B, and then againſt 1 towards the left hand upon B, is the Decimal Fraction ſought.

So $\frac{1}{4}$ will be found equal to this Decimal (*viz.*) .25: For,

As 4 upon A, is to 1 upon B; *So* is 1 upon A, to .25 upon B.

Alſo $\frac{21}{28}$ will be .75; for *As* 28 upon A, is to .21 upon B; *So* is 1 upon A, to .75 upon B.

In like manner this Vulgar Fraction (*viz.*) $\frac{582}{960}$ is equal to this Decimal, *viz.* .60625; for, ſet 960 upon A, to 582 upon B, then againſt 1 upon A, is .60625 upon B; and ſo for any other.

Problem V.

A Decimal Fraction being given, to reduce the ſame into the known Parts of the Integer.

1. IF the Decimal be part of a Pound Sterling: Set 1 upon B, to the Number of Shillings, Pence or Farthings contained in a Pound, upon A, then ſeek the Decimal given upon B, (towards the left hand) and againſt it you have the Shillings, Pence or Farthings reſpectively contain'd in the Decimal given.

Example: *Suppoſe* .7625 *were to be reduced into Shillings.*

Set 1 upon B, to 20 (the Number of Shillings in a Pound) upon A, then againſt .7625 upon B, (towards the left hand) is 15.25, that is 15 *s.* and .25 parts of a Shilling, which is equal to 3 *d.*

2. To reduce the Decimal given into Pence: Set 1 upon B, to 240 (the Pence in a Pound) upon A, then againſt .7625 toward the left hand upon B, is 183, the Pence contained in .7625.

3. But if the Decimal aforeſaid were to be reduced into Farthings?

Set 1 upon A, to 960 (the Farthings in a Pound) upon B, and then againſt .7625 upon A, is 732; and ſo many Farthings are contained in .7625.

Again, ſuppoſe .7625 were a Decimal of an Ale-Barrel, to be reduced into Gallons and Pints.

Set 1 upon A, to 32 (the Gallons in an Ale-Barrel) upon B, then againſt .7625 upon A, is 24.4, that is, 24 Gallons, and .4 tenth parts of a Gallon; to reduce this .4 into Pints; ſet 1 upon A, to 8 upon B, then againſt 4 upon A, is 3.2 Pints: So .7625 of an Ale-Barrel is equal to 24 Gallons, 3 Pints and 2 tenth parts of a Pint.

Problem VI.

Three Numbers being given, to find a fourth in a direct Proportion.

THis is called the *Rule of Three Direct*, and by the Inſtrument is wrought thus;

Set the firſt Number given upon B, to the ſecond upon A, and then againſt the third Number given upon B, is the fourth Number ſought.

Ex-

Example: *If* 8 *Quarters of Malt will make* 20 *Barrels of Strong-Beer, how many Barrels of ſuch Beer will* 22 *Quarters make?*

Set 8 upon B, to 20 upon A, and then againſt 22 upon B, is 55 upon A; and ſo many Barrels will 22 Quarters make, and according to this proportion 24 will make 60 Barrels, 28 will make 70, 32 Quarters will make 80 Barrels, *&c.*

Problem VII.

To three Numbers given, to find a fourth in an inverſed Proportion.

THis is called the *Rule of Three Inverſe*, in which obſerve, that if the *third* Number be *greater* than the *firſt*, then will the *fourth* be *leſs* than the *ſecond*. And contrariwiſe, if the *third* Number be *leſs* than the *firſt*, the *fourth* will be *greater* than the *ſecond*; and in either Caſes the Rule is this:

Set the third Number upon A, the firſt (being of like Denomination) upon B, and then againſt the ſecond Number upon A, you have the fourth upon B.

Example: *If* 8 *Men do any piece of Work in* 9 *Days; In how many Days can* 12 *Men do the ſame Work?*

Set 12 upon A, to 8 upon B, then againſt 9 upon A, is 6 upon B, which is the Anſwer. For 12 Men may do the ſame Work in 6 Days, that 8 Men will do in 9 Days.

But if the Queſtion had been, *In how many days can ſix Men do the ſame Work?* The Anſwer will be 8 2 3 For,

As 6 upon B; is to 8 upon A, *So is* 9 upon B, to 12 upon A.

Problem VIII.

Betwixt two Numbers given, to find a Mean Geometrically Proportional.

SET one of the Numbers given upon C, to the ſame Number upon D, and then againſt the other given Number upon C, is the Geometrical Mean ſought.

Example: *Let the Numbers given be* 50 *and* 72, *to find a Geometrical Mean,* &c.

Set 50 upon C, to 72 upon D, and then againſt 72 upon C, is 60 upon D. So 60 is a Geometrical Mean betwixt 50 and 72. Or thus:

Set 72 upon C, to 72 upon D, and then againſt 50 upon C, is 60 upon D, the Mean as before.

By the Pen thus:

Multiply one Number by the other, then extract the Square-Root of the Product, this Square-Root is the Geometrical Mean between the two Numbers given; thus 72 Multiplied by 50, is 3600, whoſe Square-Root is 60, the Geometrical Mean between 50 and 72.

Pro-

Problem IX.

To find the Square-Root of any Number under 1000000.

THE *Extraction of Roots* is one of the *hardeſt Leſſons* in *Arithmetick*, yet by the help of this Inſtrument it may be perform'd with *leſs trouble* than any of the foregoing Problems; For if the Lines C and D, be apply'd one to another, ſo as 10 at the end of D, be even with 10 at the end of C; I ſay, the Lines thus apply'd are like a Table ſhewing the Square Root of any Number by Inſpection only; for againſt any Number upon C, you have the Square-Root thereof upon D, *& contr.*

Note, When the Number given conſiſts of 1, 3, 5, or 7 places of Integers, ſeek it in the firſt Radius on the Line C, and againſt it you have the Root requir'd upon D.

Example: *Let the Number given be* 144; *I find this on the firſt Radius on the Line* C, *and againſt it is* 12, *the Root ſought.* In like manner,

Againſt { 576 / 20736 / 1000000 } *in the firſt Radius upon* C, *is* { 24 / 144 / 1000 } *the Root upon* D.

2. When the Number given conſiſts of 2, 4, 6, or 8 Places of Integers, find it in the ſecond Radius upon the Line C, and againſt it you have the Root upon D.

Ex-

Example: *Let* 16 *be the Number given.*

I ſeek 16 in the ſecond Radius upon C, and againſt it is 4, the Root upon D. In like manner,

Againſt	36.25	in the 2d. Rad. upon C, is	5.5	the Root upon B.
	2304		48	
	784996		886	

Problem X.

To find the Cube-Root of any Number under 1000.000.000.

PLace the Lines D and E, ſo as 10 at the end of D, be even with 10 at the end of E, then againſt any Number upon E, you have the Cube-Root thereof upon D. *Et Contr.*

Note, 1. When the Number given conſiſts of 1, 4, or 7 places of Integers, find it in the firſt Radius of the Line E, and againſt it you have the Cube-Root ſought. Example: Let the Number given be 3375: I ſeek this in the firſt Radius in the Line E, and againſt it I find 15 upon D, which is the Cube-Root of 3375, and ſo is 212 the Cube-Root of 9528128.

2. When the Number given conſiſts of 2, 5, or 8 places of Integers, find it in the ſecond Radius upon E, and againſt it is the Root ſought. Example: Suppoſe 35.937 were propounded, find this in the ſecond Radius upon E, and againſt it is 3.3, the Cube-Root upon D; in like manner is 275 the Cube-Root of 20796875.

3. When

3. When the Number given consists of 3, 6, or 9 places of Integers, seek it in the third Radius, &c. and against it is the Cube-Root: Thus against 125 in the third Radius upon E, I find 5 the Cube-Root, and so likewise is 888 the Cube-Root of 700.227.072.

Lastly, to know how many places of Integers must be in the Cube-Root of any Number given.

Put a Point over the place of Units in the Number given, then omitting 2, point every third Figure toward the left hand, then tell how many Points, for so many places of Integers must the Cube-Root consist of.

SECT. II.

Of a Superficies.

1. A *Superficies* is a Figure incompassed about with a Line or Lines, and is either *Round* or *Angular*.

2. A *Round Figure* is that which is contained by one Round Line, and is either a *Circle* (as *Fig.* 1.) or an *Ellipsis* (as *Fig.* 2.)

3. An *Angular Figure* is that which doth consist of three or more Angles, from the Number of which they are denominated, as a Figure of three Angles is called a *Triangle*; of four, a *Quadrangle*, &c.

4. A *Triangle* is a Superficies comprehended by three Right Lines (as *Fig.* 3.) of these there are six Varieties, which I shall forbear to mention, they being all measured by one and the same Rule.

5. A

5. A *Quadrangle* is a Figure comprehended by four Right-Lines, and is either a *Parallelogram* or *Trapezium*.

6. A *Parallelogram* is a Figure whose opposite Sides are parallel, having equal distances from one another in all places, and is either Right or Oblique.

7. A *Right-Angled Parallelogram*, is that whose Angles are all Right, and is either a *Square* (as Fig. 5.) or an *Oblong* (as Fig. 4.)

8. The *Oblique-Angled Parallelogram*, is that whose Angles are all Oblique, and is either a *Rhombus* (as Fig. 7.) or a *Rhomboides* (as Fig. 6.)

A *Trapezium* is a Quadrandular Figure, in which no two Sides are equal and parallel (as Fig. 8.)

10. *Figures* consisting of *more* Sides than *four*, are almost innumerable, but are reducible unto *two* sorts, *Regular* and *Irregular*, either of which are also called *Polygons*.

11. *Regular Polygons*, are such, whose Sides and Angles are equal; they take their Names from the Numbers of their Sides, as that of five Sides is called a *Pentagon* (as Fig. 9.) that of six Sides, an *Hexagon*, &c. Of *Irregular Polygons* 'tis needless to say any thing, they being measured after the same manner as a *Trapezium*.

I should next proceed to shew the Use of the Rule in the *measuring* of Lines, Superficies, and Solids: But in order to this I must premise,

1. That every *Magnitude* must be measured by some *known* kind of *Magnitude* that is *Homogeneal* (or like) to it. A *Line* is measured by a *Line*, as one Lineal Inch, Foot or Yard, &c.

A *Superficies* is measured by a *Superficies*, as one Square Inch or Foot, &c. A *Solid* is measured by a *Solid*, as one Cubick Inch, one Cubick Foot, &c.

And

And when it is known,

How many { Lineal / Square / Cubical } Inches or Feet are contained in a { Line, / Superficies, / Solid,

Then is the Quantity or Content of either of these kind of Magnitudes said to be known.

2. That the Measures of *Capacity* commonly used in *England* are of two Sorts, one for wet Commodities, as Beer, Cyder, Wine, &c. the other for dry Commodities, as Corn, &c.

And, I here take it for granted, that there are three several Gallons, whose Contents are allowed to be as followeth,

Ale, Wine, Corn } Gallon { 282 / 231 / 272¼ } Solid Inches.

Now from hence are desired the following Tables, shewing how many Solid Inches are contained in any of the other Measures respectively contained in them.

A Table of Beer-Measure.

Inches.	*Pints.*	*Quarts.*	*Gallons.*	*Firkins.*		
35 $\frac{1}{2}$						
70 $\frac{1}{2}$	2					
282	8	4				
2538	72	36	9			
5076	144	72	18	2		*Kilderkins.*
10152	288	144	36	4	2	*Barrel.*

A Table of Ale-Measure.

Inches.	*Pints.*	*Quarts.*	*Gallons.*	*Firkins.*		
35 $\frac{1}{2}$						
70 $\frac{1}{2}$	2					
282	8	4				
2256	64	32	8			
4512	128	64	16	2		*Kilderkins.*
9024	256	128	32	4	2	*Barrel.*

By an Act of the First of *William* and *Mary*, 34 Gallons is the Barrel, both for Beer and Ale, in all places, except within the weekly Bills of Mortality.

A Table of Wine-Measure.

Inches.							
28 7/9	*Pint.*						
57 3/4	2	*Quart.*					
231	8	4	*Gallon.*				
4158	144	72	18	*Rundlet.*			
12553	504	252	63	31 1/2	*Hogshead.*		
19404	672	336	84	42/3	1 2/3	*Tericon.*	
29106	1008	504	126	7	2	1 1/2	*Pipe.*
58212	2016	1008	252	14	4	3	2 *Tun.*

This Table shews, that in 1 *Tun* there are 2 *Pipes*, 3 *Tericons*, 4 *Hogsheads*, 14 *Rundlets*, 252 *Gallons*, 1008 *Quarts*, 2016 *Pints*, and 58212 *solid Inches*. The Tables for *Beer* and *Ale* are like this, and need no Explanation.

SECT.

SECT. III.

The Uſe of the Rule in meaſuring of Superficies, and firſt of a Circle.

THE Area or Superficial Content of a Circle is found by the Diameter or Circumference; I ſhall therefore firſt ſhew how by either of theſe to find the other.

Problem I.

The Diameter, or Circumference of a Circle, either being given, to find the other.

THE Circumference of that Circle whoſe Diameter is Unity (or 1) is* 3.141592653, but for our purpoſe 3.141592, will ſuffice: Therefore, as 1 is to 3.141592, ſo is the Diameter of any Circle to the Circumference: By the Inſtrument thus:

* Forbes's *Euclides Quadrat. Circuli. Propoſ.* 2.

Set 1 on the Line A, againſt 3.141592 on the Line B, then againſt any Diameter on the Line A, you have the Circumference on the Line B; and the contrary, thus: Againſt

These Diameters,	20 30 40 50	you have these Circumferences,	62.831 94.247 125.663 157.079

Or, Contrariwise, against

These Circumferences,	20 30 40 50	you have these Diameters,	6.366 9.549 12.732 15.915

And so of any other.

Problem II.

The Diameter of any Circle being given, to find the Area, (or any part thereof) in Inches, or in Ale or Wine-Gallons.

1. *For the whole Area in Inches.*

THE Area of a Circle is equal to the Fast, or Rect-Angle of half the Diameter into half the Circumference; that is to say, if half the Diameter be multiplied by half the Circumference, the Product will be the Area.

Thus, when the Diameter is 1, the Circumference is 3.141592, the half of this is 1.570796, which multiplied by half the Diameter (*viz.* .5) the Product will be the Area of that Circle, whose Diameter is 1, (*viz.*) .785398.

The

The Area's of all Circles are in proportion one to another, as the Squares of their Diameters (2. 12. of *Euclid.*)

Therefore, *As the Square of the Diameter of any Circle is to the Area of that Circle;*

So is the Square of the Diameter of any other Circle to the Area thereof.

In the Circle above-mentioned the Diameter is 1, and the Area .785398: Now the Square of 1 being but 1, it muſt hold: *As* 1 is to .785398, *So* is the Square of the Diameter of any Circle, to the Area thereof. So .785398 is a fixed Multiplicator; and if an Unit with Cyphers be divided by .785398, the Quotient will be 1.27324, a fix'd Diviſor; and by either of theſe fix'd Numbers the Area of any Circle may be found, either by Multiplication or Diviſion. For if the Square of any Diameter be

Multiplied } by { .785398 { Prod. } will be
Divided } by { 1.27324 { Quot. }

the Area in Square Inches, Feet or Yards, according as the Diameter was meaſur'd by Inches, &c.

But with more expedition by the Inſtrument, Thus:

Set 1 (a Diameter) upon the Line D, to .785398 (the Area thereof) upon C.

The Rule being thus ſet, the Lines are like a Table of Circles Areas to all Diameters; for againſt any Diameter upon the Line D, you have the Area thereof upon C.

Ex-

Example: *Let the Diameter be* 20.

Set 1 upon D, to .785398 upon C, and then againſt 20 upon D, is 314.159, the Area required upon C; and as the Rule now ſtands, I alſo find, that if

The Diameter is { 25 / 30 / 40 } the Area is { 490.87 / 706.85 / 1256.63 } Inches.

Contrariwiſe, when the

Area is { 300 / 400 / 500 } the Diameter is { 19.54 / 22.56 / 25.23 }

2. *For the Area in Gallons.*

The Area in Inches divided by 282 or 231, gives the Area in Ale or Wine-Gallons reſpectively, and ſo for any other Meaſure expreſſed in the former Tables; but without knowing the Area in Inches, the Area in Gallons may be found thus: Divide .785398

By { 282 / 231 } the Quotient will be { .0027851 A. G. / .0033999 W. G. }

The Quotients are the Area of a Circle, whoſe Diameter is 1, in Ale or Wine-Gallons, and are fixt Multiplicators for finding the Area of all Circles in either of theſe Meaſures; for if the Square of the Diameter of any Circle be multiplied by either of theſe Numbers, the Product is the Area in Ale or Wine-Gallons reſpectively.

If

If you would effect this by Diviſion, the ſeveral Diviſors are thus found: Multiply the Diviſor for finding the Area in Inches, *viz.* 1.27324

By { 282 } the Product { 359.05 } A. G.
{ 231 } will be { 294.11 } W. G.

The Products are the Diviſors ſought. And the Square of the Diameter of any Circle divided by one of theſe, quotes the reſpective Area.

Having thus found the ſeveral fixed *Multiplicators* and *Diviſors* for finding the Area of a Circle in Inches and Gallons, I ſhall here repreſent them all together in the following Table.

	Multiplicators.	Diviſors.
Inches.	.785398	1.27324
Ale-Gallons.	.0027851	359.054
Wine-Gallons.	.0033999	294.118

But the Area of any Circle may be more readily found by the help of certain fixt Numbers, called *Gage-Points*, and theſe fixt Numbers are the Diameters of thoſe Circles, whoſe Content at one Inch deep is equal to the reſpective Gallon to which they belong; thus the *Gage-Point* for the Ale-Gallon is 18.95, which is the Diameter of that Circle, whoſe Content is 282, the Square Inches in the Ale-Gallon.

The ſeveral *Gage-Points*, are the Square Roots of the Dividers laſt mentioned, and by the Rule are all found at once by the Lines C and D, for (ſetting theſe Lines even at the end)

Againſt

Againſt theſe Diviſ. upon C, { 359.05 / 294.11 } *are theſe* Gage-Points *upon* D, { 18.95 / 17.15 } A. G. W. G.

To which are ſet the Letters *a. g.* and *w. g.* upon the Rule.

Now by theſe Gage-Points the Area of any Circle may be thus found:

For Ale-Gallons,

Set 18.95 (*the Gage-Point for* Ale-Gallons) *upon* D, *to* 1 *upon* C, *then againſt any Diameter upon* D, *you have the Area upon* C.

So if the Diameter were 40 Inches, the Area will be 4.45 Ale-Gallons: And the Rule being thus ſet, the Lines are in effect a Table of Circles Areas: For I likewiſe find, that if the Diameter

be { 45 / 50 / 52.5 / 58.6 } the Area will be { 5.64 / 6.96 / 7.67 / 9.56 } A. G.

The like for Wine-Gallons, by the proper *Gage-Point.*

Note, When the Area of any Circle is ſought in Ale-Gallons, if the Diameter be more than 18.95, and leſs than 100, ſet the *Gage-Point* upon D, to 1, at the beginning of C; then againſt any Diameter, from the *Gage-Point* to 100, upon D, you have the Area upon C. Thus the Rule being ſet for Ale-Gallons, you will find that when the

Di-

Diamet. is	the Area will be	Gallons.
30	2.507	
40	4.456	
60	10.026	
80	17.825	
100	27.851	

When the Diameter is leſs than the *Gage-Point*, or more than 100, then ſet the *Gage-Point* to 1, in the middle upon C; then againſt theſe

Diameters upon D,	you have theſe Area's upon C,	A. G.
15	.627	
13	.471	
10	.278	

And without moving the Rule, if 10 at the beginning of the Line D, be 100, the Area againſt it will be 27.851: So againſt this Diameter, *viz.* 200, the Area is 111.4 Gallons, againſt 300, it is 250.7, and ſo on to 600 Inches Diameter, againſt which you have 1002.6, *&c.* by this you may further obſerve, that if the Diameter be increaſed by Tens, the Area will increaſe by Hundreds: Thus,

the Diameter	gives	Gallons.
10	.27851	
100	27.851	
60	10.026	
600	1002.6	

The like for Wine-Gallons by the proper *Gage-Point*.

Laſtly, To find any part of the Area of a Circle, in Ale or Wine-Gallons.

Set the *Gage-Point* to $\frac{1}{3}$, $\frac{1}{2}$, or any other part of 1; then againſt the Diameter you have the like part of the Area.

Ex-

Example: *Let it be required to find the third part of the Area of a Circle in Ale-Gallons.*

Set the *Gage-Point* to $\frac{1}{3}$ of 1, *viz.* .333, then againſt any Diameter you have $\frac{1}{3}$ of the Area; thus againſt 100 is 9.283, which is $\frac{1}{3}$ of 27.85, the whole Area.

Underſtand the like for Wine-Gallons, Ale or Beer-Barrels.

Problem III. Fig. 2.

The two Diameters (c d *and* e f) *of an Ellipſis, being given, to find the Area or Content in Ale-Gallons.*

AS the Square of the Diameter of a Circle is to the Area of that Circle; ſo is the Rect-Angle or Product of the greater and leſſer Diameters of an Ellipſis, to the Area thereof. Therefore multiply the greater Diameter by the leſſer Diameter, then that Product multiplied or divided by the fixed Multiplicators or Diviſors given (in pages 46 and 47,) gives the Area in Inches, Gallons or Barrels, according to the Number made uſe of.

Or thus, (by Problem VIII. Sect. I.) find a Geometrical Mean Proportion between the greater and leſſer Diameters, for this Means is the Diameter of a Circle, whoſe Area is equal to the Area of the Ellipſis.

Example. Fig. 2.

Let the greater Diameter *c d* be 72 Inches, and the leſſer *e f*, 50; by the foregoing Rule, the Geometrical Mean between them will be found to be 60, the Diameter of a Circle equal to the *Ellipſis*, and the Area of a Circle, whoſe Diameter is 60, will be found to be 10.02 Ale-Gallons. But the Area of an *Ellipſis* may be more eaſily found by the Rule, thus;

Set 359.05 upon B, to one of the Diameters (ſuppoſe 50) upon A; then againſt the other Diameter 72 upon B, you have the Area upon A, which in this Example will be 10.02, Ale-Gallons, the Content of this *Ellipſis* at one Inch deep: The like may be done for Wine-Gallons, if inſtead of 359.05, you take 249.11.

Problem IV.

To find the Area or Content of a Triangular Superficies.

IN *all Right-lined Triangles, Multiply half the longeſt Side by the Perpendicular* (which is always the neareſt Diſtance from the Baſe to the oppoſite Angle;) *or Multiply half the Perpendicular by the whole Baſe; the Product will be the Area required.*

Example. Let Fig. 3. repreſent a Triangular Back or Cooler, whoſe longeſt Side *a b*, is 260 Inches, and the Perpendicular Line *c o*, 110 Inches, the half of *a b* is 130, this multiply'd by 110, gives 14300 for the Area in Square Inches, and this divided

vided by 282, quotes 50.7, the Area in Ale-Gallons; or by the Rule at one Operation, thus:

Set 282 upon B, to 130 upon A; then againſt 110 upon B, is 50.7 upon A, the Content as before.

To meaſure Quadrangular Figures.

Problem V.

In any Right-angled Parallelogram, be it Square or Oblong: The Product of any two Sides, including one and the ſame Angle, is equal to the Area or Content.

1. *Example.*

IN Fig. 5. the Sides *b i* and *i k*, are equal; ſuppoſe each be 138 Inches; this multiply'd by it ſelf, is 19044, the Area in Square Inches: For the Area in Gallons by the Rule:

Set 282 *upon* B, *to* 138 *upon* A; *then againſt* 138 *upon* B, *is* 67.53, *the Area in Ale-Gallons at one Inch deep.*

2. Example. *Of an Oblong* (Fig. 4.)

Suppoſe *n s*, 130 Inches, and *s u*, 180, the Product of theſe (being multiply'd) is 23400, the Area in Inches: But for Ale-Gallons, ſet 282 upon B, to 180 upon A; then againſt 130 upon B, is 82.97 Ale-Gallons, the Area required.

Problem VI.

In any Oblique-Angled Parallelogram (be it a Rhombus or a Rhomboides) Multiply the ſhorteſt Diſtance between the two longeſt Sides, by one of the ſaid Sides, the Product is the Area in Inches.

1. Example. *Of a Rhombus (Fig. 7.)*

SUppoſe the Side *p r*, were 130 Inches, and the diſtance *q y*, 108, theſe multiplied, the Product will be 14040, the Area in Inches: But by the Rule thus:

As 282 is to 108, *So* is 130 to 49.78, *the Area in Ale-Gallons at one Inch deep.*

2. Example. *Of a Rhomboides* (*Fig.* 6.)

Let the Side *x z*, be 260, and the Diſtance *t w*, 108, the Product of theſe is 28080, the Area in Inches. By the Rule:

Set 282 *upon* A, *to* 260 *upon* B; *then againſt* 108 *upon* A, *is* 99.57, *the Area in Ale-Gallons.*

Pro-

Problem VII. Figure 8.

To find the Area or Content of a Trapezium.

A Right Line drawn from one of the Acute Angles to the Angle oppoſite (as the Line *a b*) will divide the Trapezium into two Triangles, the Area's of which are equal to the whole Trapezium, and may be found by the 4th Problem of this Section: Or thus; Multiply the Sum of the two Perpendiculars (*f e* and *c r*) by the half of *a b*, the Product is the Area of the Trapezium. This is ſo plain, that it needs no Example.

All other irregular Right-lined Figures conſiſting of more than four Sides, muſt be divided into Triangles (which will be ever leſs by two than the Number of Sides) and then the Area's of all thoſe Triangles are equal to the Area of the whole Figure.

Problem VIII.

The ſides of any Regular Polygon being given, to find the Area.

IN any Regular Polygon, multiply half the Sum of the Sides by the Perpendicular (or neareſt Diſtance from the Center to one of the Sides) the Product is the Area.

Example: In a Pentagon (or Figure of five equal Sides) as Fig. 9.

Suppoſe each Side be one Inch, to find the Perpendicular (C A) we have given AB = 1,5, and the Angle at C — 36 *d.*

Therefore,

	Log.
As the *Sine* of C = 36 *d.*	*Co. Ar.* 0 230781
Is to the Side A B = .5	0 698970
So is the *Sine* of B = 54 *d.*	9.907957
To the Side C A = .68819	.837708

which is the Perpendicular ſought; and this .68819 multiplied by half the Sum of the Sides, *viz.* 2. 5, the Product is 1.72047, the Area of a Pentagon, whoſe Side is one Inch; and by this method I find the Area of all the other Polygons which are expreſt in the following Table. In the firſt Column of which you have the Names of the Polygons; in the ſecond, their Area's in Inches; each of which being divided by 282, quotes the Area in Ale-Gallons, which are the Numbers in the third Column; and in the 4*th* and 5*th* Columns you have the reſpective Diviſors.

Names

Names of the Polygons.	Multiplicators. *Being the Area's in Inches and Gallons when the Side is one Inch.*		Diviſors. *Being the Square of the Sides when the Area is one Inch or one Gallon.*	
	Inches.	Ale Gall.	Inches.	Ale Gall.
Trigon	.433013	.0015355	2.309401	651.251
Tetragon	1.000000	.0035461	1.000000	282.000
Pentagon	1.720477	.0061010	.581234	163.908
Hexagon	2.598076	.0092130	.384900	108.542
Heptagon	3.633959	.0128864	.275182	77.601
Octagon	4.828427	.0171221	.207107	58.404
Nonagon	6.181824	.0219214	.161764	45.618
Decagon	7.694209	.0272844	.129968	36 651

Now having the Area (both in Inches and Ale-Gallons) of theſe Polygons when the Side is Unity, the Area of each may be readily found, the Side being given: For,

As the Square of the Side of any Polygon, is to the Area of that Polygon;
So is the Square of the Side of any other like Polygon, to the Area thereof: Therefore,

If the Area of any Polygon (expreſſed in the Table) be deſired, in Inches or Ale-Gallons.

Multiply or divide the Square of the Side by the reſpective Multiplicator or Diviſor, the Product or Quotient will be the Area ſought.

Example: For the Area in Inches, ſuppoſe the Side of a Pentagon be 50 Inches; the Square is

2500. This multiplied by 1.720477, or divided by .581234, gives 4301.2, the Area in Inches.

For the Area in Ale Gallons, If the Square of 50 (*viz.* 2500) be multiplied by .006101, or divided by 163.908, the Product or Quotient will be 15.252, the Area ſought.

To find the Side by the Area given.

Multiply or divide the given Area by the proper Multiplicator or Diviſor; the Product or Quotient will be the Square of the Side, whoſe Square-Root is the Side ſought.

Thus the Area of a *Pentagon* is 15.252 Ale-Gallons. This multiplied by 163.908, or divided by .006101, gives 2500, whoſe Square-Root is 50, the ſide ſought.

But the Lines C and D, will effect this with much more expedition; for at once ſetting of the Rule you have the Area in (Inches or Ale-Gallons) to any given Side, or the Side to any Area given.

Example: For the Area in Ale-Gallons, ſet 1 upon D, to (the Area in Gallons when the Side is 1, *viz.*) .006101 upon C; then againſt any Side upon D, you have the Area upon C, and the contrary.

Thus againſt 50 upon D, is 15.252. the Area in Ale-Gallons; Alſo againſt 22 Gallons, an Area upon C, you have 60 the Side upon D, *&c.*

SECT.

SECT. IV.

The Uſe of the Rule in the Menſuration of Solids, and firſt of a Priſm.

A Priſm *is a Solid contained by ſeveral Planes, two of which being oppoſite, are called the Baſes, and theſe are equal, Parallel alike, and alike ſituate; but the other Planes are Parallelograms, in which a Right Line may be every where applied from Baſe to Baſe:* Under this Name *Priſm* is comprehended that Solid of two Circular Baſes, uſually called a *Cylinder.*

Problem I.

To find the Solid Content of a Priſm.

MUltiply *the Area of the Baſe by the Perpendicular Height, the Product is the Content ſought.*

Example: *Let* Fig. 11. *repreſent a* Brewer's *Tun, in the form of a Square* Priſm, *whoſe Baſe* g, h, k, i, *is equal to Fig.* 5. *each Side being* 138 *Inches, and the Perpendicular* (s i) 30 *Inches, how many Ale Gallons or Beer-Barrels will this Tun contain?*

Set 282 upon B, to 138 upon A; then againſt 138 upon B, is 67.534 upon A, the Content in Ale-Gallons at one Inch deep; this multiplied by 30, the whole Depth, gives 2026.02, the whole Content in Ale-Gallons; this divided by 36, quotes 56.283, the Content in Beer-Barrels.

The Fraction .283 (being reduced) is 10 Gallons, and .188 parts. For,

As 1.000 : 36 : : .283 : 10.188.

2. Example. *Of a Round Priſm or Cylinder.*

Let Fig. 12. repreſent a round Tun, whoſe Diameter (*n o*) at Top is equal to (*s y*) the Diameter at Bottom, each being 120 Inches, and the Altitude (*e x*) 36 Inches: How many Gallons or Beer-Barrels may this Tun contain? Firſt, for Gallons.

Set the *Gage-Point* (*a g*) to 36, the Tun's Depth upon C, then againſt 120 the Diameter, upon D, is 1443.6, the Content in Ale-Gallons.

2. For the Content in Beer-Barrels: 1443.6 divided by 36, quotes 40.1, the Content in Barrels.

Suppoſe I come to this Tun, and find the Liquor 9 Inches deep: How many Gallons are then contained in the Tun?

Set the *Gage-Point* to 9; then againſt 120 is 360.95 Gallons.

Pro-

Problem II.

The Diameter, Depth and Content of any Cylindrical Tun, any two being given to find the Third.

IN this Problem are three Queſtions, all reſolved at once ſetting the Rule.

1. By the Depth and Content, to find the Diameter.

Example: *Suppoſe the Depth* 40 *Inches, and the Content* 1800 *Ale-Gallons. What is the Diameter?*

Set 40 the Depth upon C, to the *Gage-Point* upon D: Then againſt 1800 the Content upon C, is 127.1 Inches, the Diameter ſought.

2. By the Diameter and Content, to find the Depth, without moving the Rule: Say,

As 127.1 the Diameter,
is to 1800 the Content;
So is the *Gage Point*,
to 40 the Depth

} upon { D C D C }

3. By the Depth and Diameter to find the Content, the Rule ſtanding as before: Say,

As the *Gage-Point*,
is to 40 the Depth;
So is 127.1 the Diameter,
to 1800 the Content

} upon { D C D C }

Note,

Note, As a Circle is the Baſe of a Cylinder or round Priſm; ſo a Triangle, Quadrangle, or any other plain Superficies, may repreſent the Baſe of a Priſm; for if there be Planes erected perpendicularly upon the Lines which incompaſs any ſuch Superficies, they will generate a Solid which may be called a Priſm; and the Content of any ſuch Solid is gotten by multiplying the Area of the Baſe by the Altitude or diſtance from one Baſe to another.

SECT. V.

The Uſe of the Rule in the Menſuration of a Sphere, and its Fruſtums.

Problem I.

The Diameter of a Sphere or Globe being given in Inches, to find the Content thereof in Inches or Gallons, both Ale and Wine.

A Sphere is two third Parts of a Cylinder, whoſe Diameter and Altitude are equal to the Diameter of the Sphere.

But if the Diameter and Altitude of a Cylinder be 1, the Area of the Baſe, which is alſo the ſolid Content (for 1 doth not multiply) is .785398: Therefore $\frac{2}{3}$ of .785398, *viz.* .523598, is the Content of a Sphere, whoſe Diameter is Unity or 1.

More-

Moreover, all Bodies are in proportion one to another, as the Cube of their like Sides. Therefore,

As the Cube of the Diameter of any Sphere, is to the Content of that Sphere;
So is the Cube of the Diameter of any other Sphere, to the Content thereof.

Now if the Diameter be 1, the Cube thereof is but one: Therefore,

As 1 is to ·523598;
So is the Cube of the Diameter of any Sphere to the Content.

Example. *Let the Diameter of a Sphere be* 20 *Inches, and the Content required in Solid Inches.*

By the Lines D and C, on the Rule, the Cube of 20 will appear to be 8000; this multiplied by ·523598, is 4188·78, the Content ſought.

For the Content in Gallons.

If .523598 be divided

By { 282 / 231 } quotes { .0018567 / .002266 } the Content in { A. G. / W. G. }

Now if the Cube of the Diameter of any Sphere be multiplied by either of theſe Numbers, the Product will be the Content in Ale or Wine-Gallons reſpectively: Thus, the Cube of 20 the Diameter (*viz.*) 8000, multiplied by .001856, is 14.848 the Content in Ale-Gallons; and the like may be done for Wine.

But

But the Content of any Sphere may be more readily found by the Inſtrument. Thus,

Set 1 upon D, to .523598 upon E; then againſt any Diameter upon D, is the Content upon E: So againſt 20 (the Diameter above-mentioned) upon D, is 4188.78, the Content in Inches, as before: In like manner, if the Diameter were 30, the Content will be 14137.14, *&c.*

For Ale-Gallons thus:]

Set 1 upon D, to .001856 upon E; then againſt any Diameter upon D, is the Content upon E; as againſt 20 upon D, is 14.84, the Content as before; and without moving the Rule, I find that if the

Diameter { 30 } the Content { 50.13 } A. G.]
be { 40 } is { 118.78 }

Pro-

Problem II.

Having the Altitude of the Fruſtum *of a Globe, together with the Diameter of its Baſe, to find the Altitude of the other* Fruſtum; *or (which is all one) the Remainder of the Globe's Axis.*

The RULE *is,*

Divide the Square of the Semidiameter of the Fruſtum's *Baſe, by the Altitude of either of the* Fruſtums, *the Quotient will be the Altitude of the other* Fruſtum.

Example. In Fig. 10. let *c b*, the Altitude of the *Fruſtum*, be 6 Inches, and *d c e*, the Diameter at the Baſe, 24, the half of this (*viz. d c*) is 12, which Squared, is 144, this divided by the Altitude *b c*, *viz.* 6, the Quotient is 24, equal to *c a*, the Altitude of the other *Fruſtum*. Note alſo, that a Geometrical Mean Proportion between *a c* and *c b*, is equal to *c d*, which doubled, is *d e*, the Diameter of the *Fruſtum*'s Baſe.

Pro-

Problem III.

Having the Altitude of the leſſer Fruſtum *of a Sphere, and the Diameter of its Baſe, to find the Content in Inches or Gallons.*

MUltiply the Square of half the Diameter of the Baſe by three times the Altitude; and to this Product add the Cube of the Altitude, the Sum multiply'd

By { .523598 / .0018567 } Gives the Content in { Inches. / Ale-Gall.

Example: In Fig. 10. let *d c e*, the Diameter of the leſſer *Fruſtum* (*c d h e*) be 24 Inches, and *c d* its Altitude, 6 Inches; What is the Content in ſquare Inches?

c e ſquared =	144
3 times *b c* =	18
	1152
	144
	2592
Cube of *b c* =	216
	2808

2808 multiplied

By	gives	Content.
.52398		1470.263, Inc.
.0018567		5.2137, A. G.

The

The Content of the leſſer *Fruſtum* taken from the Content of the whole Sphere, leaves the Content of the greater *Fruſtum.*

SECT. VI.

The Uſe of the Inſtrument in the Menſuration of Pyramids *and their* Fruſtums.

A Pyramid *is a Body contained under ſeveral Planes, ſet upon one Right-lin'd Baſe, and meeting in a Point at the Top, which is called the* Vertex, *alſo in each of theſe Planes a Right Line may be every where applied from the Baſe to the* Vertex.

And under this Name *Pyramid* is comprehended that Pyramidical Body (whoſe Baſe is a Circle) commonly called a *Cone.*

Problem. I.

To find the Solid Content of a Pyramid.

A *Pyramid* is one third part of a *Priſm*, which hath the ſame Baſe and Altitude. Therefore.

Multiply the Area of the Baſe by $\frac{1}{3}$ of the Altitude, or $\frac{1}{3}$ of the Area by the whole Altitude, in either Caſes the Product will be the Content of the Pyramid.

Ex-

1. Example.

In a ſquare Pyramid (as Fig. 17) ſuppoſe the ſide of the Baſe *x p*, be 112 Inches, and the Altitude *? s*, 90 Inches, what is the Content in Ale-Gallons?

Set 282— upon B to 112 is 44.482 upon A,
then againſt 112 upon B

the Area of the Baſe in Ale-Gallons. Then ſet 1 upon A, to 30 upon B, and againſt 44.482 upon A, is 1334.46, the Content in Ale-Gallons.

2. Example.

There is a round Pyramid or Cone (as Fig. 18.) ſuppoſe the Diameter at the Baſe *o n*, be 100 Inches, and the Altitude *n x*, 180; How many Ale-Gallons may this Cone contain?

Set the *Gage-Point* for an Ale-Gallon, to $\frac{1}{3}$ of the Altitude (*viz.* 60) upon C: then againſt 100 upon D, is 1671, the Content required.

Pro-

Problem II.

To find the Solid Content of the Fruſtum *of a* Pyramid.

1. *BY the Rules aforegoing, find the Area of the greater, and alſo of the leſſer* Baſe.
2. *Find a Geometrical Mean Proportion betwixt theſe two Area's.*
3. *The Sum of theſe three Numbers being multiplied by one third part of the Altitude, the Product is the Content of the* Fruſtum *ſought.*

Example. Let *a c e e* (in Fig. 18.) repreſent the Fruſtum of a round *Pyramid* or *Cone*, *a o* the Diameter of the greater Baſe is 108 Inches, *e e*, the Diameter of the leſſer Baſe, 81, and *n v*, the Altitude, 36 Inches, to find the Content in Ale-Gallons.

1. Set the *Gage-Point* to 1 in the middle of the Line C; then againſt 108 upon D, is 32.48, the Area of the greater Baſe in Ale-Gallons.

2. Set the *Gage-Point* to 1 at the beginning of the Line C; then againſt 81 upon D, is 18.27, the Area of the leſſer Baſe.

3. Set 32.48 upon C, to 32,48 upon D; then againſt 18.27 upon C, is 24.36, the Geometrical Mean between the two Area's.

So the Area of the greater Baſe is	32.48
The Area of the leſſer is	18.27
A Geometrical Mean betwixt the 2 Area's is	24.36
The Sum is	75.11

This

This multiplied by $\frac{1}{3}$ of the Altitude (*viz.* 12.) gives 901.32, the Content of the Fruſtum ſought.

Or by the Pen thus:

1. Square the Diameter of the greater Baſe.
2. Square the Diameter of the leſſer Baſe.
3. Multiply one of theſe Diameters by the other. To this Product add the two Squares, and multiply the Sum by the Altitude: The laſt Product, divided by 1077, quotes the Content in Ale-Gallons.

SECT. VII.

The Uſe of the Rule in Gauging of Brewer's Tuns.

THeſe Veſſels are in ſeveral Forms: The moſt uſual may be conſidered under ſome of theſe Varieties.

I. *Such whoſe Baſes are equal.*

And if the equal Baſes be alike (that is, both Round, both Square, *&c.*) and Parallel, and the Sides of the Tun ſtreight from one Baſe to the other, theſe Tuns are called *Priſms.* See Problem I. of thi. Section.

II. *Such*

II. *Such whoſe Baſes are unequal.*

1. If the unequal Baſes be alike (ſuppoſe both Rectangular Parallelograms, both Elliptical, *&c.*) Parallel, Proportional, and alike ſituate, and the Sides of the Tun ſtreight, ſuch Tuns are the Fruſtums of Pyramids. See Problem II. of this Section.

2. If the unequal Baſes be unlike, ſuppoſe one Round, and the other Elliptical, or if they be alike but not proportional, or if they be both alike and proportional, yet if they be not alike ſituate, the Tun is called a *Priſmoid*, in which 'tis ſtill ſuppoſed the Baſes are parallel, and Sides of the Tun ſtreight. See Problem VII.

Note, When the Baſes are proportional, it will be,

As the Length of the greater Baſe, is to the Length of the leſſer:

So is the Breadth of the greater Baſe, to the Breadth of the leſſer.

And the Baſes are ſaid to be alike ſituate, when the Length of the greater and leſſer Baſes are both in the ſame Plane.

Pro-

Problem I. Fig. 11.

There is a Tun whoſe Baſes are both Equal, Parallel, and Square, each Side of either Baſe being 138 *Inches, and the Depth* 33 *Inches; what is the Content of this Tun in Ale-Gallons?* Anſwer, 2228.62. *For,*

SET 282 upon B, to 138 upon A; then againſt 138 upon B, you have 67.534 upon A, the Content at one Inch deep; this multiplied by the whole Depth, *viz.* 33, gives 2228.62, the Content required. Or thus:

Set 16.79 upon D, to 33 (the Tun's Depth) upon C, then againſt 138 upon D, is 2228.62 upon C, the Content as before.

Note, 16.79, is the *Gage-Point* for ſquare Veſſels, it being the Square-Root of 282, the Inches in an Ale-Gallon.

It is needleſs to give more Examples of this ſort of Tuns; for let the Baſes be in any Form whatſoever, if the Tun have the Qualifications above-mention'd, the Content is found by multiplying the Area of the Baſe by the Depth; and if the Baſes are round, as in moſt Tuns they are, the Content may be found at one Operation, as hath been ſhewn in the I. and II. Problem of Sect. IV.

Of

Of Tuns, whoſe Baſes are unequal.

Problem II. Fig. 16.

There is a Tun (*as* a, b, c, d, e, f, g, h,) *whoſe Baſes are Rectangular Parallelograms, Unequal but Parallel, and alike ſituate, and the Sides Proportional* (d a) *the Length of the greater Baſe, is* 100 *Inches, and* (d c) *the Breadth* 80; (h e) *the Length of the leſſer Baſe* 85, *and* (h g) *the Breadth* 68, *and the Depth of the Tun* 30 *Inches: How many Ale-Gallons may this Tun contain?*

FOR all Tuns of this ſort, this is a general Rule.

Take the Area of the greater Baſe, the Area of the leſſer Baſe, and a Geometrical Mean Proportional betwixt the two Area's; the Sum of theſe three multiplied by one third part of the Tun's Depth, gives the Contents.

Example either by the Pen, or the Lines A and B upon the Rule.

As

As 282 is to *d a* 100; *So* is *d e* 80 to the Area of the greater Baſe, *viz.*	28.368
As 282 is to *b e* 85: *So* is *b g* 68 to the Area of the leſſer Baſe,	20.496
A Geometrical Mean between theſe two Area's will be found (the Prob. VIII. Sect. 1.) to be	24.112
The Sum is	72.976

This multiplied by one third part of the Tun's Depth, *viz.* 10, gives 729.76 the Content in Ale-Gallons.

Problem III. Fig. 16.

There is a Tun whoſe Baſes (i l k m *the greater, and* n q o p *the leſſer*) *are both Elliptical, Parallel, Proportional, and alike ſituate; the greater and leſſer Diameters of each Baſe being equal to the Lengths and Breadths of the Baſes of the former Tun, and the Depth the ſame; How many Ale-Gallons will this Tun contain?*

THis Elliptical Tun may be conceived to be inſcribed in the Tun, whoſe Content was found in the laſt Problem; all the given Dimenſions are the ſame in both, and the Operation will be the ſame too; for the firſt Term in the Proportion we take 359.05 inſtead of 282. See the Work.

As

As 359.05 is to *i k* 100 ; *So* is *l m* 80 *to the Area of the greater Base.* —— } 22.281

As 359.05 *is to n o* 85 ; *So* is *p q* 68 *to the Area of the lesser Base* —— } 16.098

The Geometrical Mean between these two Area's will be found (by Prob. VIII. Sect. I.) *to be* —— } 18.938

The Sum is —— 56.317

This multiply'd by one third part of the Depth, *viz.* 10, gives 573.17 the Content of this Elliptical Tun, in Ale-Gallons.

Problem IV.

There is a Tun (as Fig. 17.) *whose Bases are both Square, but unequal* (x p) *the Side of the greater Base, is* 108 *Inches,* (r o) *the Side of the lesser Base* 93, *and the Depth,* (s a) 30 *Inches, what is the Content in Ale-Gallons ?*

THIS Tun being the Frustum of a Pyramid may be Gauged by the General Rule in Problem II. of this Section. Or thus :

1. *Square the Side of the greater Base, and also the Side of the lesser Base.*

2. *Multiply the greater Side by the lesser, and to the Product add the two former Squares.*

3. *The Sum of these three being multiplied by* $\frac{1}{3}$ *of the Depth, gives the Content in Square Inches, which divided by* 282, *the Quotient is the Content in Ale-Gallons.*

Example: By the Instrument the Square of $x\,p$ 108 is ——— 11664

The Square of $o\,r$ 93 is ——— 8649
108 multiplied by 93, is ——— 10044

Sum is ——— 30357
Multiplied by $\frac{1}{3}$ Altitude ——— 10

The Product, *viz.* ——— 303570

This divided by 282 quotes 1076.48. the Content sought.

Pro-

Problem V. Figure 18.

There is a Conical Tun whoſe Baſes are both Circular, (a o) *the Diameter of the greater Baſe is* 108 *Inches,* (c e) *the Diameter of the leſſer Baſe* 93, *and* (u n) *the depth* 30 *Inches: What is the Content of this Tun in Ale-Gallons?*

THE Dimenſions being the ſame as in Problem IV. of this Section, proceed as is there directed, the laſt Product will be 303570, this divided by 359.05, quotes 845.48 the Content ſought.

Or thus by the Rule.

Subſtract the Diameter of the leſſer Baſe from the Diameter of the greater, and add half the Difference to the leſſer, the Sum is the Diameter in the middle of the Tun's Depth; which found,

1. Set the *Gage-Point* (*a g*) to the Tun's depth upon C.

Then againſt the Diameter upon D, is a fourth number upon C, which keep.

2. Set the *Gage-Point* to $\frac{2}{3}$ of the Tuns depth upon C.

Then againſt $\frac{1}{2}$ the Difference of the Diameters upon D, is a fourth Number, which added to the fourth Number firſt found, is the Content of the Tun ſought.

Example. Figure 18.

c o the Diameter of the greater Baſe is	108
c e the Diameter of the leſſer Baſe is	93
The Difference is	15

The half Difference is 7.5, this added to 93 makes 100.5 for the Diameter in the middle of the Tuns depth: *Therefore,*

1. Set the *Gage-Point* to 30 the Tuns Depth, then againſt 100.5 the Diameter is 843 91, which keep.

2. Set the *Gage-Point* to $\frac{1}{3}$ of the Tuns Depth, that is, 10; then againſt 7.5 the Semidifference of the Diameters, is 1.56. This added to 843.91 (the Number firſt found) the Sum is 845.47; the Content ſought; which agrees very well with the former Rule.

Problem VI.

To find the Content of a Pyramidal Tun whoſe Baſes are in the Form of any of the firſt 10 *Regular Polygons, one Side of each Baſe, and the Tuns Depth being given.*

THE Area of each Baſe of any ſuch Tun may be found by Problem VIII. Section III.

A Geometrical Mean betwixt theſe Area's may be found by Problem VIII. Section I.

The Sum of the two Area's and the Geometrical Mean between them being multiplied by $\frac{1}{3}$ of the Tuns Depth, gives the Content. Or thus:

By

By the Rule in the laſt Problem, ſquare the ſide of each Baſe, multiply the greater Side by the leſſer; then multiply the Sum of theſe three by $\frac{1}{3}$ of the Tuns Depth. Laſtly, multiply or divide this laſt Product by the proper Multiplicator or Diviſor [*as per* Table page 55.] the Product or Quotient will be the Content in Ale Gallons.

Example. Let there be a Pyramidal Tun, whoſe Baſes are both in the form of a *Pentagon* (or Figure of five equal ſides) and let the ſide of the greater Baſe be 108, the ſide of the leſſer 93, and the depth 30 Inches; all the ſame as in the laſt Problem: Therefore according to the Rule and Example there laid down, the laſt Product will be 303570: This multiplied by .006101, or divided by 163.908, [*vid.* Table, page 55.] gives 1852.08.

To find the Content of any Tun, whoſe Parallel Baſes are Rectangular Parallelograms, or Ellipſes, the ſides of the Tun being ſtreight from the Top to the Bottom, whether the Baſes be alike or unlike, proportional or diſproportional, alike ſituated or inverted, by one general Rule.

Problem VII.

Let Fig. 16. *repreſent a Tun called a Priſmoid, the Baſes are Rectangular Parallelograms, unequal and diſproportional, but parallel and alike ſituate;* (a d) *the Length below* = 150 *Inches, and* (a b) *the Breadth* 100, (e h) *the Length above* 90, *and* (e f) *the Breadth* 70, *and the depth* 30: *What is the Content in Ale Gallons?*

The RULE.

1. *TO the greater Length* (a d) *add* $\frac{1}{2}$ *the leſſer Length* (e h) *and multiply the Sum by the greater Breadth* (a b) *reſerving that Product.*

2. *To the leſſer Length* (e h) *add* $\frac{1}{2}$ *the greater Length* (a d) *and multiply the Sum by the leſſer Breadth* (e f) *add this Product to the former reſerved Product.*

3. *Multiply the Sum of theſe two Products by* $\frac{1}{3}$ *of the Tun's Depth, and divide the laſt Product by* 282, *the Quotient will be the Content of the Tun in Ale-Gallons.*

Example. (*a d*) 150 added to $\frac{1}{2}$ (*e b*) 45, the Sum is 195; this multiply'd by (*a b*) 100, the Product is 19500, which keep.

2. (*e b*) 90 added to $\frac{1}{2}$ (*a d*) 75, the Sum is 165; this multiplied by (*e f*) 70, the Product is 11550; this added to the former Product, *viz.* 19500, the Sum

Sum is 31050; this multiplied by $\frac{1}{3}$ of the Depth, *viz.* 10, the Product is 310500, and this divided by 282, gives 1101.06 the Content in Ale-Gallons.

The truth of this Rule will plainly appear, if we duly consider Fig. 13, in which (*a b c d*) is equal to the greater Base, and (*e f g b*) equal to the lesser Base of the Tun in the last Problem (which was represented by Fig. 16)

In this, Figure 13, (*b i o v*) is made equal to (*f e b g*).

Also (*a b*) 100 less by (*e f*) 70, is equal to (*a i*) 30.

And (*a d*) 150 less (*s b*) 90, is equal to (*s d*) 60, the Altitude is 30 as before.

The Lines being drawn, the whole Solid is composed of these parts, *viz.* three Prisms and a Pyramid.

The $\left\{\begin{matrix}1\\2\\3\end{matrix}\right\}$ Prism is $\left\{\begin{matrix}b\ i\ o\ u\ f\ e\ b\ g\\ i\ a\ s\ o\ b\ e\\ o\ v\ c\ n\ b\ g\end{matrix}\right\}$.

The Pyramid is *o s n d b.*

The Content of these four Solids being added together, gives the Content of the whole Solid *a b c d e f g b*; for the whole is equal to all its parts taken together.

And the Content of these may be found thus:

1. Multiply (*f g*) 90 by (*f e*) 70, the Product is 6300; this multiplied by (*b f*) the Altitude, *viz.* 30, gives 189000 the Content of the Prism, *b i o v f e b g.*

2. Multiply (*a i*) 30 by (*a s*) 90, the Product is 2700; this multiply'd by $\frac{1}{2}$ of the Altitude, *viz.* 15, is 40500, the Content of the Prism *i a s o b e.*

3. Multiply (*e n*) 70 by (*n o*) 60, the Product is 4200; this multiplied by 15, is 63000, the Content of the Prism *o v c n b g.*

4. Multiply (*s d*) 60 by (*s o*) 30, the Product is 1800; this multiply'd by $\frac{1}{3}$ of the Altitude, *viz.* 10, gives 18300, the Content of the Pyramid *s o n d b.*

So the Content of the 1, 2, 3 Prism is		189000
		40500
		63000
Pyramid is		18000
The Sum		310500

Is the Content in square Inches, and is exactly the same with the Content found by the Rule given in the last Problem.

The Truth of this Rule being thus demonstrated, we shall shew its further Usefulness in the following Problems.

Prob-

Problem VIII. Figure 16.

There is a Tun (called a Cylindroid) whoſe Baſes are both Elliptical, unequal, and diſproportional (that is, there is not the ſame Proportion betwixt the Axis *and Diameter of the greater Baſe, as is betwixt the* Axis *and Diameter of the leſſer) let the given Dimenſions be the ſame as in the laſt Problem,* viz.

$n\,o = 90 : q\,p = 70$ } Rectangular } above
$i\,k = 150 : l\,m = 100$ } Conjugates } below.

And the Depth 30 *Inches: What is the Content of this Tun in Ale-Gallons?*

For the Content of this Tun, work by the General Rule, and according to the Example given in the laſt Problem, only inſtead of 282, divide the laſt Product by 359.05. So in this Example (the Numbers being the ſame as in the laſt) the laſt Product will be 310500, this divided by 359.05 quotes 864.78, the Content in Ale-Gallons.

Prob.

Problem IX. Figure 15.

There is a Tun whose Bases are Rectangular Parallelograms, and Parallel, but neither equal nor alike situate (for g h, the Breadth above, is posited directly opposite to a d, the Length below) how many Ale-Gallons will this Tun contain?

The Dimensions are,

b g Breadth = 68 : *b e* Length = 85 above.
d a Length = 100 : *d c* Breadth = 80 below.
The Depth 30 *Inches.*

If the Bases of this Tun were alike situate (as is supposed in Problem II. of this Section) the Content would be 729.76 Ale-Gallons, as was there shewn; but the Bases being inverted as aforesaid, the Content will be found to be 735.81 Ale-Gallons, as appears by the following Work, which is performed by the General Rule laid down in Problem VII. of this Section.

Thus (*b g*) 68 added to $\frac{1}{2}$ (*d a*) 50, is 118; this multiplied by (*b e*) 85, the Product is 10030.

Again (*d a*) 100 added to $\frac{1}{2}$ (*b g*) 34, is 134, this multiplied by (*d c*) 80, the Product is 10720, the Sum of these two Products is 20750; which multiply'd by $\frac{1}{3}$ of the Depth, *viz.* 10, gives 207500, the Content in square Inches; this divided by 282, the Quotient is 735.81 the Content in Ale-Gallons.

Or if the last Product be divided by 359.05 the Quotient will be 577.91 the Content of an Elliptical

tical Tun, whoſe Axis above is directly oppoſite to the ſhorter Diameter below; and ſuch an Elliptical Tun may be conceived to be inſcribed in the other, the reſpective Axis's and Diameters, Lengths, and Breadths, and alſo the Depth being the ſame in both.

Problem X. Figure 14.

To find the Content of a Tun whoſe Baſes are unlike, ſuppoſe one Square, and the other Oblong; or one Round, and the other Elliptical, but Parallel one to the other, and the ſides of the Tun ſtreight.

The Dimenſions are,

Breadth.	Length.		
$e f = 74$:	$f g = 100$	Rectangular	above.
$d c = 100$:	$c b = 100$	Conjugates	below.

Depth 30 *Inches.*

By the general Rule in Problem VII. Work thus:

1. To $(e f)$ 74 add $\frac{1}{2}$ $(d c)$ 50, the Sum is 124, this multiplied by $f g$ 100, is 12400.

2. To $(d c)$ 100 add $\frac{1}{2}$ $(e f)$ 37, the Sum is 137, this multiplied by $(c d)$ 100 is 13700; this added to the former Product, *viz.* 12400, makes 26100, and this multiplied by $\frac{1}{3}$ of the Depth, *viz.* 10, the Product is 261000; this divided by 282, gives 929.53, the Content of this Tun in Ale-Gallons, ſuppoſing the Baſes were one Square, and the other Oblong, as the Figure repreſents them.

But if the Baſes were one Round, and the other Elliptical, the laſt Product, *viz.* 261000 muſt be divi-

divided by 359.05, and in this caſe the Content will be 726.91.

By what hath been ſaid, I preſume it will not be difficult to find the whole Content of any Tun, whoſe Baſes are parallel, and Sides ſtreight from Top to Bottom.

SECT. VIII.

Shewing how to find the Content of any Tun from Inch to Inch, or what any Tun will contain upon every Inch of its Depth, which is commonly call'd Inching of a Tun.

Problem I.

Suppoſe a Tun whoſe Baſes are both Round, or both Square and Parallel one to another, and in the Sides a ſtreight Line may be every where apply'd from Baſe to Baſe: Let the Diameter, or Side of the greater Baſe be 187.2 *Inches, the Diameter or Side of the leſſer Baſe* 180, *and the Depth* 12 *Inches, to Inch this Tun (be it Round or Square) from the leſſer Baſe downward:*

RULE.

FRom the Side or Diameter of the greater, ſubtract the Side or Diameter of the leſſer Baſe, divide the Difference by the Tuns Depth, and call the Quotient (*a*). For

For the Side or Diameter of the leſſer Baſe, put (*b*).

Multiply the Number (*b*) by the Number (*a*) and call the Product (*c*). Then,

1. To the Square of (*b*) add the Number (*c*), and one third part of the ſquare of (*a*), the Sum of theſe three will ſhew the Content of the firſt Inch of the Tun.

2. Multiply one third part of the Square of (*a*) by 2; to this Product add the Number (*c*); then multiply the Sum by 2, and to this Product add the Square of (*b*); multiply this Sum by 2, this laſt Product is the Content of the two firſt Inches of the Tun.

3. Multiply one third part of the ſquare of (*a*) by 3; then add and multiply as in the laſt, the third and laſt Product will ſhew the Content of the three firſt Inches of the Tun.

4. Having got theſe three Numbers, ſubtract the firſt from the ſecond, and likewiſe the ſecond from the third; then the firſt Number, and each of theſe Differences or Remainders being ſeverally divided by 282 for ſquare Tuns, or by 359.05 for round Tuns, the reſpective Quotients will be the Content (in Ale-Gallons) of the firſt, ſecond and third Inches of the Tun, by which a Table may be made for the whole Tun, as in the Example following.

Example of a Square Tun.

The Side of the great Baſe	187.2
The Side of the leſſer Baſe	180.0
The Remainder is	7.2

This divided by the Tuns Depth, *viz.* 12. quotes .6 for the Number (*a*).

The

The Number *b* (which is the ſide of the leſſer Baſe) is 180 ; this multiply'd by the Number *a*, *viz*. .6, gives 108 for the Number *c*.

1. The Square of *b* is	32400
Number *c* is	108
$\frac{1}{3}$ of the Square of *a* is	.12
The Sum is the Content of the firſt Inch of the Tun, in Inches, *viz*.	32508.12
2. $\frac{1}{3}$ of the Square of *a* is	.12
Multiply by	2
Product	.24
Add the Number *c*	108.00
Sum is	108.24
Multiply by	2
Product	216.48
Add the Square of the Number *b*	32400.00
Sum is	32616.48
Multiply by	2
Product is the Content of the two firſt Inches of the Tun, in Inches, *viz*.	65232.96

3.

3. $\frac{1}{3}$ of the Square of *a*	.12
Multiply by	3
Product	.36
Add the Number *c*	108.00
Sum is	108.36
Multiply by	3
Product	325.08
Add the Square *b*	32400.00
Sum is	32725.08
Multiply by	3
Product is the Content of the three first Inches of the Tun, in Inches,	98175.24
Two first Inches	65232.96
First Inch	32508.12
Rest for second Inch	32724.84
Three first Inches	98175.24
Two first Inches	65232.96
Rest for third Inch	32942.28

Now

Now theſe three Numbers.

viz. { 32508.12 / 32724.84 / 32942.28 } Divided by 282, quotes { 115.277 / 116.045 / 116.816 }

For the Content (in Ale-Gallons) upon the firſt, ſecond, and third Inches of this Tun, ſuppoſing the Baſes ſquare.

And if the ſaid Numbers be divided by 359.05, the reſpective Quotients will

be { 90.539 / 91.142 / 91.747 } the Content of the firſt, ſecond and third Inches of the Tun, when its Baſes are Round.

Now, by theſe Numbers the following Table is compoſed for the Round Tun. Thus,

Take the firſt Inch from the ſecond, the Remainder or Difference is .603, which ſet down in the third Column, againſt the ſpace betwixt the firſt and ſecond Inches. In like manner take the ſecond Inch from the third, the Remainder is .605, which place in the third Column under the other, then take .603 from .605, there remains .002 for the ſecond Difference, which is always the ſame, as in the fourth Column, this ſecond Difference added to the ſecond Number in the third Column *viz.* .605, makes 607, and this added to the Content of the third Inch, *viz.* 91.747, makes 92.354, the Content of the fourth Inch; and ſo by a continual Addition of the ſecond Difference to the firſt, and of their Sum to the Inch laſt found, the Table is made.

Depth

Depth. Inches.	*Content of every Inch.* in Ale Gall.	*First* Differ.	*Second* Differ.
1	90.539	.603	
2	91.142	.605	.002
3	91.747	.607	.002
4	92.354	.609	.002
5	92.963	.611	.002
6	93.574	.613	.002
7	94.187	.615	.002
8	94.802	.617	.002
9	95.419	.619	.002
10	96.038	.621	.002
11	96.659	.623	.002
12	97.282		
Sum — 1126.766 — Content of the Tun.			

In this Example we have made a Table for the Round Tun, and the Operation would have been the ſame for the Square Tun, had we taken the three firſt Inches of *that* inſtead of *theſe*.

Note, When we Inch a Tun by this Rule, we ſuppoſe it to be Regular, having all the Qualifications expreſt in this Problem. But foraſmuch as moſt Tuns have ſome Irregularity or Unevenneſs in their Sides, I ſhall here lay down a more Practical Rule for Inching of Tuns.

Pre-

Problem II.

There is a Tun whoſe Diameter at the Bottom is 108 *Inches, the Diameter at the top* 93, *and the Depth* 30 ; *how many Ale-Gallons will this Tun contain upon every Inch of the Depth ?*

TO reſolve this Queſtion, there muſt firſt be known the whole Content of the Tun, and alſo the Area of the mean Diameter in the middle of every 10 or 12 Inches of the Depth, and theſe may be thus found.

Take (with ſome convenient Inſtrument) a Diameter in the middle of every 10 Inches of the Tun's Depth, and ſuppoſe in this Example the firſt (at 5 Inches from the Top) be 95.5, the ſecond (at 15 Inches from the Top) 100.5, and the third 105.5, theſe I ſet down as in the following Table.

This done, we may conſider the Tun either as Round or Square, and firſt to Inch it as a Round.

Set the *Gage Point* (*ag*) to 1 ; then againſt the ſeveral Diameters already found, you have their reſpective Area's : So the firſt Area will be 25.40, the ſecond 28.13, the third 31.00 ; theſe I ſet down in the Table againſt their Diameters, and having added them together, the Sum is 84.53, this multiplied by 10 (or which is all one, remove the Prick one place towards the Right-hand) the Product is 845.3, the whole Content of the Tun.

Note,

Note, Here the Diameters being taken in the middle of every 10 Inches, there will be no uſe of Multiplication or Diviſion in Inching the Tun, as there will be when the Diameters are taken in the middle of every Foot or half Foot, for here the Area's of the ſeveral Diameters do not only ſhew the mean Inch in the middle of every 10 Inches, but alſo the whole Content of thoſe 10 Inches, if you remove the Prick but one place more towards the Right-hand. Thus the mean Inch of the laſt 10 Inches is 31.00, but if the Prick be removed as aboveſaid, it is 310, which is the Content of the Tun at 10 Inches deep; I therefore reduce the ſeveral Contents in the third Column into Barrels, Firkins, and Gallons, Beer-meaſure, and ſet them down in the fourth Column, theſe added together are 23 Barrels, 1 Firkin, 8 Gallons, and 3 tenths of a Gallon, which agrees with the Sum of the Numbers in the third Column.

Parts of the Tun's Depth.	*Mean Diameters in the middle of every 10 Inch. of the Tuns Depth.*	*Mean Area's or Contents of every 10 Inch. of the Tuns Depth.*	*Content of every 10 Inches Beer-Meaſure.* B.	F.	G.
10	95.5	25.40	2	0	2 0
10	100.5	28.13	7	3	2.3
10	105.5	31.00	8	2	4.0
30		84.53	23	1	8.3

By theſe Numbers thus found it will be eaſie to make a Table, which ſhall ſhew the quantity of Liquor contain'd in the Tun at any depth, for which obſerve the following Directions.

1. Set

1. Set down the whole Content of the Tun in Barrels, Firkins, Gallons, and Parts, in their reſpective Columns, and at the top of the firſt Column againſt the Content ſet 0, and under it the Tuns Depth in Inches, 1, 2, 3, 4, 5, *&c.* this done take the firſt mean Inch, *viz.* 25.4, and reduce it into Barrels, Firkins, and Gallons, Beer-Meaſure, it makes 0 : 2 : 7 : 4 : ſet this upon a ſcroll of Paper, and ſubtract it continually from the whole Content till you come at the tenth Inch of the Tuns depth, which done, if you miſtake not, the Remainder there will be 26 : 1 : 6 : 3.

2. Take the ſecond mean Inch, *viz.* 28, 13, this being reduced as the former is 0 : 3 : 1.13, ſubtract this continually from the laſt Remainder, till you come at 20 Inches deep, and there your Remainder will be 8, 2, 4.

Take the third and laſt mean Inch, and reduce and ſubtract it as before, and if your Work be right, there will be no remainder at the laſt Inch or Bottom of the Tun; now the Table being made, you may caſt away the Fractions as uſeleſs. The uſe of this Table is obvious to the meaneſt Capacity, without any further Explanation:

In.	P.	F.	G.
0	23	1	8.30
1	22	3	0.9
2	22	0	2.5
3	21	1	4.1
4	20	2	5.7
5	19	3	7.3
6	19	0	8.9
7	18	2	1.5
8	17	3	3.1
9	17	0	4.7
10	16	1	6.3
11	15	2	5.17
12	14	3	4.04
13	14	0	2.91
14	13	1	1.78
15	12	2	0.65
16	11	2	8.52
17	10	3	7.39
18	10	0	6.26
19	9	1	5.13
20	8	2	4.00
21	7	3	0
22	6	3	5
23	6	0	1
24	5	0	6
25	4	1	2
26	3	1	7
27	2	2	3
28	1	2	8
29	0	3	4
30	0	0	0

For

For if you come to this Tun, and find 2 Inches of the Depth dry, the quantity of Liquor then in the Tun is 22 B. 0 F. 2 G. If 7 Inches be dry, there remain in the Tun but 18, 2, 1, *&c.* Thus for Round Tuns.

Now, ſuppoſe the Tun laſt mention'd had been Square, and the Sides at Bottom and Top equal to the Diameters of the laſt, and the Depth the ſame; the Diameters or Sides in the middle of every 10 Inches of the Tuns depth will be the ſame as in the laſt; by which the ſeveral mean Inches, and alſo the whole Contents of every 10 Inches are found to be as in this Table.

Parts Tuns Depth.	*Sides of the Tun in the middle of every* 10 *Inches.*	*Mean Area's or Content of every ten Inches.*	*Content of every* 10 *Inches Beer Meaſure.*		
			B.	F.	G.
10	95 5	32.34	8	2	8.4
10	100.5	35.81	9	2	6.1
10	105.5	39.46	10	3	7.6
30		107.61	29	2	4.1

For, ſet 282 upon B, to 95.5, the firſt Diameter upon A, then againſt 95.5 upon B, is 32.34 the firſt mean Inch, and after the ſame manner find the ſecond and third, which being reduced as in the laſt Example, I find the whole Content to be 1076.1 Ale-Gallons (or which is the ſame thing) 29 B, 2 F, 4.1 G. this being done, you may proceed to Inch this Tun by the ſame directions which were given in the laſt Example.

Pro-

Problem III.

To Inch a Tun whoſe Baſes are unlike, viz. the lower Circular, and the upper Elliptical, ſuppoſe the Diameter of the Circular Baſe be 100 *Inches, the Axis (or longeſt Diameter) of the Elliptical Baſe* 100, *and the ſhorteſt* 90, *and the Tun's Depth* 20 *Inches.*

TO Inch this Tun, I take the croſs Diameters in the middle of every 4 Inches of the Tun's depth: Thus at 2 Inches from the Top I find the longeſt Diameter 100, and the ſhorteſt 91, in the middle of the next four Inches (which is 6 Inches from the Top) the longeſt Diameter is 100. and the ſhorteſt 93. and thus I find the reſt, which are as in this Table is expreſs'd: Now having theſe Diameters, the Area or mean Inch in the middle of every 4 Inches of the Tun's Depth may be thus found.

Set 359.05 upon B, to the longeſt Diameter, (*viz.* 100) upon A, then againſt the ſhorter Diameter (*viz.* 91) upon B, is 25.34, the firſt mean Inch, or Area in Ale-Gallons; and after the ſame manner the ſecond and third will be found, as in the following Table.

Depth

Depth.	Axis.	Shorter Diam.	Mean Inch. in Ale-Gallons.	Content of every 4 Inches of the Tuns Depth.
4	100	91	25.34	2 3 2.36
4	100	93	25.88	2 3 4.52
4	100	95	26.47	2 3 6.88
4	100	97	27.02	3 0 0.08
4	100	99	27.57	3 0 2.28
0			132.28	14 . 2 . 7.12

Now if the Mean Inches be severally multiplied by 4, and these Products reduced into Barrels, Firkins, and Gallons, Beer-measure, they will give the Numbers in the last Column, which added together are 14 Barrels, 2 Firkins, 7 Gallons, and 12 parts of a Gallon.

Set this down for the first Number of your Table, and then reduce the first mean Inch into Barrels, Firkins, and Gallons, and substract it 4 times from the whole Content and Remainders; do the like with the other mean Inches, and so complete the Table as in the last Example.

Problem I.

To find the Drip or Fall of a Tun, and to make Allowance for the same in Inching the Tun.

Example.

ADmit the Tun last mentioned were so placed, that when the Bottom is but just covered on one side, the Liquor is 4 Inches deep on the side op-

oppoſite; How much muſt be allow'd for the Fall of this Tun.

By the Operation aforegoing the Area of the mean Diameter in the middle of the 4 laſt Inches (that is, at 2 Inches from the bottom) was found to be 27.57, this multiplied by 4, is 110.28 Ale-Gallons, the half of this is 55.14, and ſo much may be allow'd for the Fall of the Tun; reduce theſe 55 14 Gallons into Barrels, Firkins, and Gallons Beer-meaſure, gives 1 : 2 : 1.14: and this taken from 14 : 2 : 7.12, the Content of the Tun when it ſtands level, there remains 13 Barrels, 0 Firkins, 5 Gallons, and 98 parts: And ſo much will the Tun contain, when the Fall is 4 Inches; therefore ſet this at the Top of the Table, then reduce the firſt mean Inch into Barrels, Firkins, and Gallons, and ſubtract it four times from the whole Content, the remainder will be 10 : 1 : 3 : 62: Do the like with the ſecond, third, and fourth mean Inches, and you will find the Remainder at 16 Inches dry, to be 1 : 1.14: And ſo much it takes to cover the Bottom, as was before obſerved.

0	1	3	C	5.98
1	12		1	7.64
2	11		3	0.30
3	11		0	1.96
4	10		1	3.62
5	9		2	4.74
6	8		3	5.86
7	8		0	6.98
8	7		1	6.10
9	6		2	8.63
10	6		0	0.16
11	5		1	0.69
12	4		2	1.22
13	3		3	1.20
14	3		0	1.18
15	2		1	1.16
16	1		2	1.14

As for ſuch Tuns as are ſuppoſed to be the Fruſtums of *Spheroids*, *Parabolick Conoids*, *Hyparabolick Conoids*, or *Parabolick Spindles*, I ſhall give Rules for Gauging and Inching of theſe in the Appendix, where I ſhall alſo endeavour to explain the nature of theſe Figures, and ſhew whence they are derived.

Pro-

Problem V. Figure 23.

To Gauge a Copper, and make Allowance for the Crown; let a, b, c, d, e, a, *repreſent a Copper whoſe Content is required.*

1. TO find the ſeveral Diameters and Depth of the Copper, together with the Altitude of the Crown.

Take a ſmall Cord, or Thread, make one end faſt at *a*, and extend the other to the oppoſite Side of the Copper at *e*, where make it faſt, or cauſe ſome Perſon to hold it very ſtraight; then ſet one End of the Inſtrument in the Bottom of the Copper at *b*, and move it to and fro, till you find the neareſt Diſtance to the Thread (as at *o*,) this Diſtance (*b o*,) is the Depth of the Copper, ſuppoſe it be 44 Inches.

In like manner ſet the End of the Rule upon the Top of the Crown at *c*, and take the neareſt Diſtance to the Thread (as *c v*,) ſuppoſe it be 36 Inches, this ſubtracted from (*b o*) 44, the Remainder 8, is the Altitude of the Crown.

To find (*b d*) the Diameter of the Bottom of the Crown.

Meaſure (*a e*) the Diameter at the Top, admit it be 115 Inches, then hold a Thread ſo as a Plummet at the End thereof may hang juſt over *b*; by this means you may find the Diſtance *a o*, and on the other Side *e s*, ſuppoſe each be 14.5, add theſe together, and ſubſtract their Sum (*viz.* 29) from *a e*, 115, the Remainder 86, is equal to *b d*, the Diameter at the Bottom of the Crown: The Diameter which touches the Top of the Crown may

be found by the Inſtrument, ſuppoſe it be 91.4 Inches.

Now, to find the Content of the Copper from the Crown upwards (that is, the part *a b c i e*) the Depth (*v c*) being 36 Inches, you may take a Diameter in the middle of every 6 Inches betwixt *v* and *c*, which ſuppoſe to be as in the ſecond Column of the following Table, the Numbers in the third Column are the reſpective Area's in Ale-Gallons, found by the *Gage-Point* as before; the fourth Column ſhews the Content of every 6 Inches in Barrels, Firkins, and Gallons Beer-meaſure: Theſe added together are 29 Barrels, 3 Firkins, 2.5 Gallons.

And ſo much will the Copper contain after the Crown is cover'd.

Now if the Crown be taken for the Fruſtum of a Sphere, the Content (by the third Problem of *Section* V.) will be found to be 83.3 Gallons.

But the Content may be more readily found, very near the truth: Thus,

The Diameter *b d* was found to be 86, the Area to this Diameter is 20.6, this Multiplied by ⅓ the Crown's Altitude, *viz.* 4, gives 82.4, the Content of the Crown, this Subſtracted from the Content of the part or portion *b c i d n b*, the Remainder is the quantity of Liquor that will cover the Crown; the Content of the part *b c i n d b*, may be thus found, *b i* is 91.4, and *b d* is 86, theſe added together are 177.4, the half 88.7 may be taken for a mean Diameter, the Area to this Diameter is 21.9, this Multiplied by 8, the Altitude, gives 175.2 the Content.

Now Subſtract 82.4 (the Content of the Crown) from 175.2, the Remainder is 92.8, or 2 Bar. 2 Fir. 2 Gal. and .8 of a Gal. this added to the Content

first

firſt found, *viz.* 29 Bar. 3 F. 2.5 Gal. gives 32 B. 1. F. 5.3 Gal. the Content of the whole Copper, as in the following Table.

Parts of the depth.	*Diameter.*	*Mean Inch.*	*Content of every 6 Inches in* B.	F.	G.
6	113.1	35.62	5	3	6.7
6	109.2	33.21	5	2	1.2
6	105.28	30.86	5	0	5.1
6	101.36	28.61	4	3	0.6
6	97.33	26.37	4	1	5.2
6	93.4	24.29	4	0	1.7
36	Sum		29	3	2.5
To cover the Crown			2	2	2.8
Content Copp.			32	1	5.3

The Content being thus found, you may proceed to Inch the Copper by the ſame Directions which were given for Inching of Tuns in the laſt Problem.

Problem VI.

The Diameter of any Round, or Side of any Square Maſh-Tun, *being given, to find how many Gallons of* Mault *it will contain at one Inch deep.*

THE Corn-Gallon (which is the Meaſure we here account by) is ſuppoſed to contain 272.25 Cubick Inches.

But Mr. *Dary*, in his compleat Gauger, ſaith, that an indifferent ſort of Mault, and of an indifferent Grinding, when three Worts ſhall have paſſed through it, will be more compact by about a ſixth part; he therefore concludes, that 227 may be a proper Diviſor for Square Maſh-Tuns, and 388 for Round ones.

1. *For Round Maſh-Tuns.*

The Diameter of that Circle, whoſe Area is 227, is 17, and this may be called the *Gauge-Point* for Round Maſh-Tuns; and to find the Content of any ſuch Maſh-Tun:

Set 17 upon D, to 1 upon C; then againſt any Diameter upon D, you have the Content in Gallons at one Inch deep: Thus if the Diameter be 113, the Area or Content at once Inch deep would be 44.3 that is, 5 Buſhels, and 4.3 Gallons.

2. *For Square Maſh-Tuns.*

If a Maſh-Tun be Square or Oblong, the Content at one Inch deep may be thus found: Say,

As 227 is to the Length;
So is the Breadth to the Content.

Example. Suppoſe the Length 112 Inches, and the Breadth 80, how many Gallons will this Tun contain at one Inch deep? Anſwer, 39.4: For,

Set 227 upon A, to 80 upon B; then againſt 112 upon A, is 39.4, the Content upon B.

SECT.

SECT. IX.

Of Cask Gauging.

BEFORE we can find the Content of a Cask, we muſt know the Dimenſions of it, *viz.*

The { Bung / Head / Length, } Diameter, } in Inches.

And when theſe are known, the form or ſhape of the Cask muſt be conſidered; for although the Diameters (aboveſaid) and the Length of one Cask may be equal to thoſe of another, yet one of theſe Casks may contain ſeveral Gallons more than the other, and therefore the Content of all Casks cannot be found by one and the ſame Rule.

Now moſt Writers in Treating on this Subject, have taken it for granted, that every common Cask (as Butt, Pipe, Hogſhead, Barrel, and ſuch like;) is in the form of one of theſe Solids, *viz.*

1. The middle Fruſtum of a *Spheroid.*
2. The middle Fruſtum of a *Parabolical Spindle.*
3. The middle Fruſtum of two *Parabolical Conoids*, abutting upon one common Baſe; or,
4. The middle Fruſtum of two *Cones*, abutting upon one common Baſe.

Accordingly they have laid down Rules for finding the Content of theſe Solids, and by thoſe

Rules they ſuppoſe the Content of any Cask may be found,

As for inſtance.

If the Dimenſions of a Cask be

Bung, Head, Length } Diameter { 32, 24 } Inches.
Length 40

By the Rules given by Mr. *Dary*, and others, the Content of this Cask if taken for

				Ale-Gall.
The middle Fruſtum of	2	*Spheroid*	is	97.455
		Parabolical Spindle.		96.505
	2	*Parabolic. Conoids*		89.136
		Cones		87.934

The Rules are true, as to the firſt and laſt of theſe Solids, *viz.*

The middle Fruſtum of { a *Spheroid*, two *Cones.*

And theſe may be repreſented by Figure 21.

A Cask which is the middle Fruſtum of a *Spheroid* is ſhewn by the outermoſt Line *d*, *o*, *a*, *d*, *f*, *c*, *o*, *b*, *o*, *c*, *e*, *d*, the two ſtraight Lines *d*, *e*, *c*, and *d*, *f*, *e*, are the Heads of the Cask, and the Curved Lines *d*, *o*, *a*, *o*, *d*, and *c*, *o*, *o*, *c*, the Staves.

If

If a Cask (can) be in the form of the middle Fruſtum of two *Cones*, it may be repreſented by the ſtraight Lines *d*, *a*, *a*, *d*, and *c*, *b*, *b*, *c*, the Heads *d*, *e*, *c*, and *d*, *f*, *c*, the Bung-Diameter *a*, *b*, and the Length *e*, *f*, are all the ſame in both theſe Casks, and yet 'tis evident by the Figure it ſelf, that one will contain more than the other, and by the Example above cited, the difference is 10.25, Gallons, for the *Spheroid* contains 95.455 Gallons, and the Fruſtum of two *Cones* but 87.934.

There are very few (if any) Casks whoſe Diameters and Length are equal to thoſe abovenamed, that will contain more than 97.455 Gallons (the Contents of the *Spheroid*) nor leſs than 87.934 (the Content of the Fruſtum of two *Cones*) ſo that all the varieties that can happen will fall betwixt theſe two, and the Staves of any Cask differing from theſe, will lie between the Lines *d*, *e*, *a*, and *d*, *u*, *a*, in Figure 21.

Now for the other two Solids, *viz.* the middle Fruſtum of a *Parabolical Spindle*, and the middle Fruſtum of two *Parabolical Conoids*; according to the Authors above-cited, the Content of the firſt is 96.505 Gallons, and the ſecond 89.136.

Whether the Rules they have given for finding theſe Contents be true or falſe, I ſhall examine in the Appendix.

But ſuppoſing they are true, yet they do not anſwer the various caſes that may happen in common Practice, for in the Example above, the Content of a Cask taken, as the middle Fruſtum of a *Parabolical Spindle*, is but .95 of a Gallon leſs than the Content of the Fruſtum of a *Spheroid*, and the Content of the Fruſtum of two *Parabolical Conoids* is but 1.931 more than the Fruſtum of two *Cones*.

Now I preſume no Man will pretend to diſtinguiſh which Notion a Cask muſt be taken in, whether as the Fruſtum of a *Spheroid*, or as the Fruſtum of a *Parabolical Spindle*, when the Contents differ not ſo much as a Gallon.

Moreover, here is above 7 Gallons Difference betwixt the Contents of the Fruſtum of a *Parabolical Spindle*, and the Fruſtum of two *Parabolical Conoids*: So that I may modeſtly affirm, that there are many Casks whoſe Content cannot be found by any of theſe Rules; for it muſt be granted, that a Cask whoſe Diameters and Length are equal to thoſe in this Example, may contain ſome certain quantity betwixt 96 Gallons, and 89.

As ſuppoſe 93, I ſay, if this Cask were to be Gauged by any of the Rules above-cited, it will give the Content at leaſt 3 Gallons too much, or 4 Gallons too little.

This premiſed, I come now to lay down ſome ſhort Rules, by which the Content of any Cask may be found very near the Truth.

Pro-

Problem I. Fig. 21.

There is a Cask ſuppoſed to be the middle Fruſtum of a Spheroid, *intercepted between two Plains Parallel, cutting the Axis at Right Angles, let* a b *the Diameter at the Bung be* 32 *Inches,* d c *the Diameter at the Head* 24, *and the Length* e f, 40, *Inches; How many Ale or Wine-Gallons will this Cask contain?*

RULE.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add half the Difference of the ſaid Squares; the Sum of theſe multiplied by the Length, and this Product divided

By { 1077 / 882 } Quotes the Content in { Ale / Wine } Gallons.

Example.

Square of *a b* 32 is	1024
Square of *d c* 24 is	576
Sum is	1600
Half Sum	800
Half the Difference of the Square is	224
Sum is	2624

Which multiplied by 40 (the Length of the Cask) gives 10496, this Divided by 1077, Quotes 97.455, the Content in Ale-Gallons.

Or divided by 882, the Quotient will be 119.002, the Content in Wine-Gallons.

Note, The Common Rule for Gauging the Fruſtum of a *Spheroid*, (by taking twice the Square of the Bung Diameter, and the Square of the Head-Diameter, &c.) is ſomewhat a ſhorter way than this: But this Rule being more agreeable to the Work in the following Problems, I have thought fit to inſert it here: The Reader may uſe which he pleaſes.

Problem II.

Suppoſe a Cask (of the ſame Diameters, and Length with the former) ſhall have its Staves betwixt the Bung and Head leſs Curved, ſo as to leave out about a third part of the ſpace between the Lines d o a, *(which repreſent the* Spheroid) *and* d u a, *as in* Figure 22. *How many Gallons will this Cask contain?*

RULE.

To the Sum and half Sum of the Squares of the Bung and Head Diameters, add three tenth parts of the Difference of the ſaid Squares; the Sum of theſe Multiplied by the Length, and the laſt Product divided by 1077, *or* 882, *Quotes the Content in Ale or Wine-Gallons reſpectively.*

Example.

Example. *The Dimenſions as above.*

Sum of the Squares is	1600
Half Sum is	800
The Difference of the Squares Multiplied by .3, is	134.4
Sum of theſe is	2534.4

This multiplied by .40, gives 101.376, and this Divided by 1077 quotes 94.128, the Content in Ale-Gallons, or by 882, the Quotient will be 114.938, the Content in Wine-Gallons.

Problem III. Figure 19.

There is a Cask whoſe Staves, between the Bung and Head, have but little Curveture, lying nearer to the ſtrait Line a u d, *than to the Curved Line* a o d, *What is the Content of this Cask in Ale or Wine-Gallons?*

RULE.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add one tenth of the Difference of the Squares; the Sum of theſe Multiplied by the Length, and this Product divided as above directed, quotes the Content in Ale or Wine-Gallons reſpectively.

Example.

Example. (*Dimenſions as above.*)

Sum of the Squares is	1600
Half Sum is	800
One tenth of the Difference is	44.8
Sum of theſe is	2444.8

Which Multiplied by 40, gives 97792, this Divided by 1077, the Quotient is 90.800, the Content in Ale-Gallons; or by 882, quotes 110.875, the Content in Wine-Gallons.

Problem IV. Figure 21.

To find the Content of a Cask in the form of the middle Fruſtum of two Cones abutting upon one common Baſe (if any ſuch can be) it may be repreſented by the ſtrait Lines d u a, a u d.

RULE.

From the Sum and half Sum of the Squares of the Bung and Head Diameters, Subtract half the Square of the Difference of the Diameters themſelves; Multiply the Remainder by the Length, and Divide this Product as in the laſt, the Quotient will be the Content in Ale or Wine-Gallons.

Example.

Example. *(Dimenſions as above.)*

Sum of the Squares is	1600
Half Sum is	800
Sum of theſe is	2400
Difference of the Diameter is 8, this Squared is 64, the half is	32
The Remainder is	2368

This Multiplied by 40, gives 94920, and this Divided by 1077 quotes 87.934, the Content in Ale-Gallons; or by 882 the Quotient will be 107.619, the Content in Wine-Gallons.

Thus it appears by the four Problems aforegoing, if the Dimenſions of a Cask

Be { Bung, Head } Diameter ——— { 32, 24 } Inches,
Length ——— 40

The Content in each variety is as followeth;

		Ale-Gall.	Diff.
By Problem	I.	97.455	
	II.	94.128	3.327
	III.	90.800	3.328
	IV.	87.934	2.066

And

And here it may be obſerved, that the Differences of the Contents thus found are very near equal: So that by theſe Rules there is not ſo much room for Error, as by the Rules which ſeveral Authors have preſcribed.

I have inſiſted the longer upon this Buſineſs, that I might prevent Miſtakes, for we cannot be too exact, where a ſmall Error muſt needs be prejudicial either to the King, or to the Merchant, eſpecially when the Duty upon Brandy is great.

Problem V.

The Dimenſion of any Cask being given, to find the Content by the Inſtrument.

TO do this, we muſt firſt find a mean Diameter, that is, ſuch a Diameter as ſhall reduce the Cask to a Cylinder; for which purpoſe I have Calculated theſe three Tables, for the three firſt varieties of Casks abovementioned.

A Ta-

A Table for Equating the Diameters of the middle Fruſtum of a Spheroid.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.67	0.73	0.80	0.87	0.94	1.01	1.08	1.15	1.22	1.29
2	1.36	1.43	1.50	1.57	1.64	1.71	1.78	1.85	1.92	1.99
3	2.06	2.13	2.20	2.27	2.34	2.41	2.48	2.55	2.62	2.69
4	2.76	2.83	2.90	2.97	3.04	3.11	3.18	3.25	3.32	3.39
5	3.46	3.54	3.61	3.68	3.75	3.82	3.89	3.96	4.03	4.10
6	4.17	4.24	4.31	4.39	4.46	4.53	4.60	4.67	4.74	4.81
7	4.88	4.95	5.03	5.10	5.17	5.25	5.32	5.39	5.46	5.53
8	5.60	5.68	5.75	5.82	5.90	5.97	6.04	6.11	6.18	6.26
9	6.32	6.40	6.47	6.55	6.62	6.70	6.77	6.84	6.91	6.98
10	7.05	7.13	7.20	7.28	7.35	7.43	7.49	7.56	7.64	7.71

A Table for the Second Variety of Casks.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.62	0.68	0.74	0.81	0.87	0.93	0.99	1.06	1.12	1.18
2	1.24	1.30	1.36	1.43	1.49	1.55	1.62	1.68	1.74	1.80
3	1.86	1.93	1.99	2.05	2.12	2.18	2.24	2.30	2.37	2.43
4	2.49	2.55	2.62	2.68	2.75	2.81	2.88	2.94	3.01	3.07
5	3.13	3.20	3.26	3.33	3.39	3.46	3.52	3.59	3.65	3.72
6	3.78	3.85	3.91	3.98	4.04	4.11	4.18	4.24	4.31	4.37
7	4.44	4.50	4.57	4.63	4.70	4.76	4.83	4.90	4.96	5.03
8	5.10	5.16	5.23	5.30	5.36	5.43	5.49	5.56	5.63	5.70
9	5.77	5.84	5.91	5.97	6.03	6.10	6.17	6.24	6.30	6.37
10	6.44	6.51	6.57	6.64	6.71	6.77	6.83	6.90	6.96	7.02

A Table for the Third Variety of Casks.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.55	0.60	0.65	0.70	0.76	0.81	0.87	0.93	0.99	1.05
2	1.11	1.17	1.22	1.28	1.33	1.39	1.44	1.50	1.55	1.61
3	1.67	1.73	1.78	1.84	1.90	1.95	2.01	2.06	2.12	2.18
4	2.24	2.30	2.35	2.41	2.46	2.52	2.57	2.63	2.69	2.75
5	2.81	2.87	2.93	2.98	3.04	3.10	3.16	3.21	3.27	3.33
6	3.39	3.45	3.51	3.56	3.62	3.68	3.73	3.80	3.86	3.91
7	3.97	4.02	4.08	4.14	4.20	4.26	4.32	4.38	4.44	4.50
8	4.56	4.62	4.68	4.74	4.80	4.86	4.92	4.98	5.04	5.10
9	5.16	5.22	5.28	5.35	5.41	5.47	5.53	5.59	5.65	5.71
10	5.77	5.83	5.89	5.96	6.02	6.08	6.14	6.20	6.26	6.32

Now by these Tables the mean Diameter of any Cask is thus found.

Subtract the Head-Diameter from the Bung-Diameter then seek the Difference in the Table, taking the whole Inches in the first Column, and the Tenths (if any be) at the Head, in the Angle of meeting you have a Number, which being added to the Head-Diameter, gives the mean Diameter sought.

Example

Example. Let the Dimenſions be as in the firſt Problem of this Section, the Difference of the Diameter is 8: In the Table for a *Spheroid*, againſt 8 on the Side, and under 0, I find 5.6, this added to the Head-Diameter *(viz.* 24) the Sum is 29.6, the mean Diameter ſought.

This being obtained, the Content of any Cask may be found by the Inſtrument at one Operation, for which the Proportion is:

As the Gage-Point is to the Cask's Length;
So is the mean Diameter to the Contents.

Example. The Length 40, and the mean Diameter 29.6 Inches.

Set the *Gage-Point* for Ale-Gallons to 40 upon C, then againſt 29.6 upon D, is 97.455, the Content in Ale-Gallons, which is the ſame as was found by the firſt Problem of this Section.

Note, The three Tables above are expreſſed in three Scales, and placed upon one Side of the Rule againſt the Line of Inches, by which the mean Diameter of any Casks may be readily found.

To

To find the Ullage of a Cask.

Problem VI.

Let Figure 21 *repreſent a Cask poſited with its Axis* e f, *Parallel to the Horizon, in part empty, the Surface of the Liquor cutting the Bung Diameter in* S, *ſuppoſe the whole Content* 97.455 *Gallons, the Bung-Diameter* a b, 32 *Inches, the wet part* s b, 24, *the dry part* S a, 8; *How many Gallons are contained in the Cask? And how many will it require to fill it full?*

FOR reſolving this, and ſuch like Queſtions, there is a Line of Segments on the Rule, with a Line of Numbers to ſlide againſt it, and upon theſe Lines the Proportion is:

1. *As the Bung Diameter upon the Numbers, is to* 100 *upon the Segments;*

So is the wet or dry parts on the Numbers, to a fourth Number on the Segments.

2. *As* 100 *upon* B, *is to the Cask's whole Content upon* A;

So is the fourth Number laſt found to the Anſwer.

Example.

Example. 1. Set 32 the Bung-Diameter on the Numbers, to 100 on the Segments, then againſt 8, the dry Inches on the Numbers, is 15.5 on the Segments.

2. Set 100 upon B, to 97.455, the Cask's whole Content upon A, then againſt 15.5 upon B, you have 15.105 upon A, and ſo many Gallons it will take to fill up the Cask; this ſubtracted from 97.455, the whole Content, there reſts 82.35, the quantity of Liquor in the Cask; which may otherwiſe be found thus, having ſet 32 the Bung-Diameter to 100 on the Segments, againſt 24 the wet Inches on the Numbers, you have 84.5 on the Segments. *Then,*

As 100 *upon* B, *is to* 97.455 *upon* A;
So is 84.5 *upon* B, *to* 82.35 *upon* A: the ſame as above.

Note, The Line of Segments on this Rule is made for a *Spheroid*, and is therefore more exact for all Bulging Casks than the Table of Segments, which are proper only for a Cylinder.

The following Tables ſhew the Ullage of two ſeveral Casks in Gallons and Quarts: They were made by drawing out the Liquor by an exact Wine-Quart, and the dry Inches taken by an Inſtrument proper for that purpoſe.

A Table shewing the Ullage of a Cask in Gallons and Quarts, the Length of the Cask is 37.3 Inches, the Bung-Diameter 26.5, the Head-Diameter 25, and the Content 100 Wine-Gallons.

Dry Inches.	Gall. & Quarts.	Dry Inches.	Gall. & Quarts.	Dry Inches.	Gall. & Quarts.
0.71	1	6.19	13 2	10.98	32 2
1.10	2	6.33	14 0	11.09	33 0
1.40	3	6.48	14 2	11.20	33 2
1.62	1 0	6.62	15 0	11.31	34 0
1.82	1 1	6.76	15 2	11.42	34 2
2.00	1 2	6.90	16 0	11.53	35 0
2.16	1 3	7.04	16 2	11.64	35 2
2.30	2 0	7.18	17 0	11.75	36 0
2.44	2 1	7.32	17 2	11.86	36 2
2.57	2 2	7.46	18 0	11.97	37 0
2.69	2 3	7.60	18 2	12.08	37 2
2.80	3 0	7.73	19 0	12.19	38 0
2.91	3 1	7.87	19 2	12.30	38 2
3.01	3 2	8.00	20 0	12.41	39 0
3.11	3 3	8.14	20 2	12.52	39 2
3.20	4 0	8.26	21 0	12.63	40 0
3.30	4 1	8.39	21 2	12.74	40 2
3.39	4 2	8.52	22 0	12.85	41 0
3.48	4 3	8.65	22 2	12.96	41 2
3.57	5 0	8.77	23 0	13.07	42 0
3.65	5 1	8.90	23 2	13.18	42 2
3.73	5 2	9.02	24 0	13.28	43 0
3.81	5 3	9.14	24 2	13.39	43 2
3.89	6 0	9.26	25 0	13.49	44 0
4.05	6 2	9.38	25 2	13.60	44 2
4.21	7 0	9.50	26 0	13.70	45 0
4.37	7 2	9.62	26 2	13.81	45 2
4.53	8 0	9.73	27 0	13.92	46 0
4.69	8 2	9.85	27 2	14.02	46 2
4.84	9 0	9.96	28 0	14.12	47 0
4.99	9 2	10.08	28 2	14.23	47 2
5.15	10 0	10.19	29 0	14.33	48 0
5.30	10 2	10.31	29 2	14.44	48 2
5.45	11 0	10.42	30 0	14.54	49 0
5.60	11 2	10.54	30 2	14.65	49 2
5.75	12 0	10.65	31 0	14.75	50 0
5.90	12 2	10.76	31 2		
6.04	13 0	10.87	32 0		

A Table ſhewing the Ullage of a Cask to every half Gallon, the Length of the Cask is 30.8 Inches, the Bung-Diameter 25.6, the Head-Diameter 22.6, and the Content 64 Wine-Gallons.

Dry Inch.	Gall. and Quart	Dry Inch.	Gall. and Quart	Dry Inch.	Gall. and Quart
.80	2	5.76	11 2	9.60	22
1.25	1 0	5.94	12 0	9.77	23
1.60	1 2	6.12	12 2	9.94	23
1.91	2 0	6.30	13 0	10.11	24
2.17	2 2	6.48	13 2	10.28	24
2.40	3 0	6.66	14 0	10.45	25
2.63	3 2	6.84	14 2	10.62	25
2.85	4 0	7.01	15 0	10.79	26
3.07	4 2	7.19	15 2	10.96	26
3.28	5 0	7.36	16 0	11.13	27
3.49	5 2	7.54	16 2	11.30	27.
3.69	6 0	7.71	17 0	11.47	28
3.89	6 2	7.89	17 2	11.64	28
4.08	7 0	8.06	18 0	11.81	29
4.27	7 2	8.2	18 2	11.98	29
4.46	8 0	8.41	19 0	12.14	30
4.65	8 2	8.58	19 2	12.31	30
4.84	9 0	8.75	20 0	12.47	31
5.03	9 2	8.92	20 2	12.64	31
5.21	10 0	9.09	21 0	12.80	32
5.40	10 2	9.26	21 2		
5.58	11 0	9.43	22 0		

T

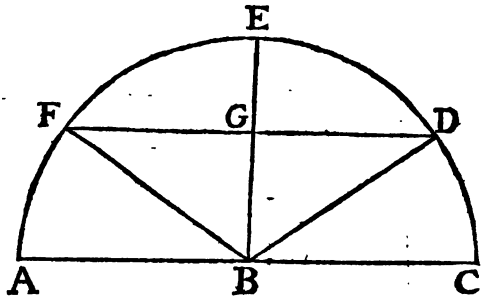
The Casks by which theſe Tables were made, were a Brandy Punchion, and a Brandy Hogſhead, both from *Bourdeaux*: The Tables will ſerve indifferently for all ſuch Casks, whoſe Diameters, Length, and Contents are the ſame with theſe, and are ſo plain, that they need no Example to ſhew their uſe; for againſt the dry Inches you have the Gallons and Quarts that the Cask wants of being full, and if the Bung-Diameter be more than half dry, enter the Table with the wet Inches inſtead of the dry, it ſhews the quantity of Liquor in the Cask: Theſe Tables may by Proportion be converted into Lines, and put upon a Rule.

By what hath been already ſaid, 'tis eaſie (I preſume) to find the Ullage of a Cask whoſe Axis is poſited Parallel to the Horizon: But leſt this Tract ſhould be thought imperfect, I ſhall here inſert a Table of the Segments of a Circle, and alſo ſhew their Conſtruction and Uſe.

A Table of the Segments of a Circl whoſe Area is Unity, the Diamet (*viz.* 1.128378) being divided into 10 equal parts.

V.	Segm.	V.	Segm.	V.	Segm.	V.	Segm.
1	.0017	99	.9983	26	.2066	74	.7934
2	.0048	98	.9952	27	.2178	73	.7822
3	.0087	97	.9913	28	.2292	72	.7708
4	.0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	.2523	70	.7477
6	.0245	94	.9755	31	.2640	69	.7350
7	.0308	93	.9692	32	.2759	68	.7241
8	.0375	92	.9625	33	.2878	67	.7122
9	.0446	91	.9554	34	.2998	66	.7002
10	.0520	90	.9480	35	.3119	65	.6881
11	.0598	89	.9402	36	.3241	64	.6759
12	.0680	88	.9320	37	.3364	63	.6636
13	.0764	87	.9236	38	.3487	62	.6513
14	.0851	86	.9149	39	.3611	61	.6389
15	.0941	85	.9059	40	.3735	60	.6265
16	.1033	84	.8967	41	.3860	59	.6140
17	.1127	83	.8873	42	.3986	58	.6014
18	.1224	82	.8776	43	.4112	57	.5888
19	.1323	81	.8677	44	.4238	56	.5762
20	.1424	80	.8576	45	.4364	55	.5636
21	.1526	79	.8474	46	.4491	54	.5509
22	.1621	78	.8369	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	.5255
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The Conſtruction of the Table of Segments of a Circle.



In the Figure here before us, all that part or portion comprehended within the Right Lines DB and BF, and the Circular Line FED, is called the Sector of a Circle, FDG is the Cord of the Segment, GE is the Verſed Sine, and GB the Complement thereof, ABC is the Diameter of the Circle, which here we ſuppoſe to be 2.

Now, if the Diameter of a Circle be 2, the Length of the Circumference is double the Area of the Circle, for the Circumference is 6.2831852, and the Area 3.1415926; hence it evidently follows, that the Length of half the Arch of any Sector of this Circle, is equal to the Area of that Sector, and (as ſhall be proved anon) the Natural Sine of the ſaid half Arch is equal to the Area of the Triangle in that Sector.

Now, the Area of the Triangle, taken from the Area of the Sector, leaves the Area of the Segment.

Example.

Example. Let it be required to find the Area of the Segment FEDG, whose verſed Sine GE is .4.

1. *To find the Area of the Sector.*

In order to which we muſt firſt know the Quantity of the Angle at the Center, and that is thus diſcovered.

Seek the Complement of the verſed Sine in the Table of Natural Sines, the Degrees and Decimal Parts (or Minutes) thereunto belonging are the Coſine of half the Angle at the Center.

Thus BE 1, — GE. 4 = BG. 6, this found in the Table of Natural Sines, gives 36 Degrees and 87 Parts, the Angle ABF, its Complement is 53 *d.* 13′. the Arch FE, which doubled is 106 *d.* 26′. the Arch of the Sector FED, that is, the Quantity of the Angle at the Centers.

Now to find the Length of any Number of Degrees, (that is, the Length of any Arch) of this Circle. Say,

As 360, *the Degrees in the whole Circle, to* 6.2831852, *the Length of the whole Circumference*;
So *is* 1, *to* .01745329, *the Length of one Degree.*

And this is a common Multiplicator for any Number of Degrees, ſo the Degrees in the Arch FE, *viz.* 53.13, being Multiplied by .01745329 gives .92727787, the Length of the Arch FE, that is, the Area of the Sector BFED, which was required.

2. *To find the Area of the Triangle* B F D.

The Diameter being .2, I ſay, half the Natural Sine of the Angle at the Center, when the ſaid Angle does not exceed 90 *d.* when it does then half the Natural Sine of its Complement to 180, is equal to the Area of the Triangle in the Sector.

Thus in the Sector BFED, the Angle at the Center is 106.26, its Complement to 180 is 73.74, whoſe Natural Sine by the Table is .96, the half of this is .48, the Area of the Triangle BFD, which was required.

The truth of which doth thus appear, BF is 1, BG is .6, *ergo* GF is .8, for BFq—BGq=FGq (by 47.1 of *Euclid.*)

But FG .8 Multiplied by BG .6 gives .48 the Area of the Triangle BFD as before, Then

From the Area of the Sector BFED	92727787
Take the Area of the Triangle BFD	48000000
There remains the Area of the Segment FEDG	44727787

But ſeeing in all Tables ſhewing the Area's of the Segments of a Circle, it is ſuppoſed the Area of the whole Circle is Unity, the foregoing Table was made by this Proportion.

As the Area of that Circle whoſe Diameter is 2, *viz.* 3.1415926, is to the Segment of any part of that Diameter:

So is 1 (the ſuppoſed Area of another Circle) to the Segment of the like part of its Diameter. Thus,

As 3.1415926, is to .4472778 the Segment laſt found,

So

So is 1 to .14237, the Segment answering to the verſed Sine 20, in the Table.

For as the verſed Sine of the firſt Segment, viz. GE was .4 that is $\frac{2}{5}$ of the Radius EE;

So 20 the verſed Sine of this laſt Segment is $\frac{2}{5}$ of the Radius of the Table.

The Uſe of the TABLE *of Segments.*

Is to find the Ullage or Quantity of Liquor remaining in a Cask, whoſe Axis is poſited Parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask, and for this the Rule is;

To the wet or dry Inches of the Bung-Diameter add a competent number of Cyphers, then divide it by the whole Diameter, the Quotient found in the Table under the Title VS. gives a Segment, which multiplied by the whole Content of the Cask, the Product ſhews the quantity of Liquor in the Cask, if the Dividend was the wet Inches, or the Ullage if it was the dry.

Example. Let there be a Cask in the form of a Cylinder, whoſe Bung-Diameter is 33 Inches, the dry part 12, and the Content of the Cask 108 Gallons: how many Gallons are wanting to fill up the Cask?

12 Divided by 33 quotes 36, which found in the Table under VS gives 32412; this Segment multiplied by 108 (the whole Content of the Cask) the Product is 35. and ſo many Gallons the Cask wants of being full.

Note, If a Cask be in the form of a Cylinder, or near that *Figure*, the Table will give the Ullage more exactly than the Line of Segments on the Rule:

but if the Bung-Diameter be much greater than the Head-Diameter, the Line of Segments is truer than the Table.

Problem VII. Figure 23.

There is a Cask ſuppoſed to be the middle Fruſtum of a Spheroid, *ſtanding upon one Head, with its Axis Perpendicular to the Horizon, the Dimenſions*

Are { E H Length——— 24 / A B Bung-Diameter— 20 / C D Head-Diameter— 16 } Inches.

To calculate any Diameter betwixt the Bung and Head (ſuppoſe S V) and by ſuch Diameter to find the Quantity of Liquor in the Cask when part is empty.

1. *To find the Diameter the Rule is,*

From the Square of the Bung-Diameter (A B) ſubtract the Square of the Head-Diameter (C D) and divide the Square Root of the Remainder by half the Cask's Length (E O); then multiply this Quotient by the Number of Inches, that the Diameter ſought is diſtant from the Bung, and call this Product your Subducend. Laſtly, from the Square of the Bung-Diameter, ſubtract the Square of your Subducend, the Square Root of the Remainder is the Diameter ſought.

Example.

Example. Let it be required to find the Diameter SV, which is ſuppoſed to be 6 Inches diſtant from AB, the Bung-Diameter.

The Square of the	Bung-Diameter is	400
	Head-Diameter is	256
The Remainder is		144

Whoſe Square-Root is 12, this divided by half the Cask's Length, *viz.* 12, the Quotient is 1, this multiplied by 6, gives 6 for the Subducend.

Square of the Bung-Diameter	400
Square of the Subducend	36
There remains	364

Which is the Square of the Diameter SV, its Square Root is 19.07878, the Diameter it ſelf; but the Square of it is ſufficient for our purpoſe.

2. *To find the Quantity of Liquor in the Cask.*

Having found the Square of SV, the Diameter of the Liquor's Surface, the Quantity of Liquor is eaſily attained:

For by the Rule laid down in the firſt Problem of this Section, the whole Content of the Cask will be found to be 23.532 Ale-Gallons, the half of this is 11.766, for the Content of the lower half, Ant B, and by the Rule now cited, the Content of the part ASVB will be found to be 6.484 Gallons.

For the Square of AB is	400
The Square of S V is	364
Sum is	764
Half Sum is	382
Half the Difference of the Square is	18
Sum of theſe is	1164

This multiplied by *o g*, *viz.* 6, gives 6984, and this divided by 1077 quotes 6.484, the Content of the part ASVB, as above; now this added to the Content of the lower half, *viz.* 11.766, gives 18.25, and ſo many Gallons are in the Cask. If this be taken from 23.532, the whole Content, there reſts 5.282, and ſo many Gallons are required to fill up the Cask: for this 5.282 Gallons, is the Content of the part SCDV.

If the Cask were leſs than half full, the Ullage might be found by the ſame method, and that ſubtracted from the whole Content leaves the Quantity of Liquor in the Cask.

SECT.

SECT. X.

Of Gauging of Worts in open Veſſels.

FOR this purpoſe there are two Lines placed on the under ſide of the two Sliding Pieces of the Rule, which being ſo plain and obvious as not to need any Deſcription, I ſhall proceed to ſhew their Uſe, which is to find how many Ale-Gallons, and Hundred parts of a Gallon, any ſmall Tub, or ſuch like open Veſſel (from 12 to 36 Inches Diameter) will contain at one Inch deep; to perform which obſerve the following Directions.

☞ Let the Sliding Piece C D, be pin'd faſt at the end towards the Right-Hand, then draw out the other Sliding Piece, 'till the two ends touch any two oppoſite ſides of the Tub or Veſſel, then look what Diviſions (both upon the Line of Gallons and Line of Inches upon the Sliding Piece) are cut by the end of the Rule, for that Point doth ſhew the Diameter of the Tub in Inches and tenth Parts upon one Line; and upon the other, how many Gallons and hundred parts it will contain at one Inch deep.

Thus when the end of the Rule cuts 13.4 on the Line of Inches, it alſo cuts 0 Gallons .5 tenths on the Line of Gallons; in like manner, when it cuts 21 on the Inch Line, it will at the ſame time cut 1.23 on the Line of Gallons; that is, 1 Gallon and .23 hundred parts; and ſo much will a Tub

of 21 Inches Diameter contains at one Inch deep. And thus you may draw out this Piece to 24 Inches and an half, and then the end of the Rule will cut 1.673 on the Line of Gallons.

Now if the Diameter of the Tub be more than 24.5, pin this Piece at 24.5, and then draw out the other Sliding Piece at the other end, where you will find, that when the end of the Rule cuts 30 Inches, it will alſo cut 2.51 Gallons; and thus you may draw out this Piece to 36 Inches, where you will find 3 Gallons, 6 tenths and more: *Note*, In all theſe Caſes 'tis the Area only which is ſought, the Inches do alſo ſhew the Diameter, but this is to make the other more plain.

Now the Tubs or Veſſels which Victuallers commonly make uſe of for cooling their Worts, and working their Ale, are generally Cylindrical or Conical, and in either of theſe the Baſes are Circular or Elliptical.

Firſt of ſuch as are Cylinders, or near that Form.

If the Baſes of a Cylindrical Tub be circular, the Rule ſhews the Area or Content at one Inch deep by Inſpection, (as above) this multiplied by the Depth of the Liquor gives the Content.

For inſtance.

Admit I come to a Tub, and (by the Rule) find the Area to be 1 Gallon 4 tenths, and the Depth of the Wort 7 Inches and an half:

Multiply

Multiply the Area ——— 1.4
by the Depth, *viz.* ——— 7.5

70
98

The Product is ——— 10.50

That is, 10. Gallons and an half, which is the Content ſought.

But this Content may be found without the Pen, for having the Area and the Depth, you may multiply the one by the other, by the Rule: Thus,

Set 1 upon B, to 1.4 the Area upon A; then againſt 7.5 the Depth upon B, you have 10.5 upon A, the Content as before.

If the Veſſel be Elliptical, take the croſs Diameters in the middle of the Depth; as ſuppoſe the longeſt Diameter is 60 Inches, and the ſhorteſt 40 ½, ſay,

As 359 is to one of the Diameters;
So is the other Diameter to the Area.

And by this Proportion on the Lines A and B, the Area of the Ellipſis above-mentioned will be found to be 6.68: For,

As 359 upon B, is to 40 upon A;
So is 60 upon B, to 6.68 upon A.

This is the true way of finding the Area of an Ellipſis. But in ſmall Elliptical Tubs you may find the Area with leſs trouble: Thus,

Find (by drawing out the Rule) the Area of the longeſt Diameter, and in like manner the Area of the ſhorteſt Diameter, half the Sum of theſe two is the Area ſought.

For Inſtance.

Suppoſe the Area of the longeſt Diameter be	3.62
And the Area of the ſhorteſt	2.51
The Sum is	6.13
The half is	3.065

The Area required, which is very near the truth; for if you work by the former Rule, you will find the true Area to be 3.008.

Secondly, for Conical Veſſels.

When the Diameters at Top and Bottom differ not above 4 Inches, you may find the Area in the middle of the Depth, by drawing out the Rule as above directed, and this Area thus found is very near a true mean for the whole Tub.

Example. Let the Diameter at the Bottom of a Conical Tub be 24 Inches, and the Diameter at the Top 28 Inches, in the middle of the Depth of this Tub you will find (by the Rule) the Diameter 26 Inches, and the Area 1.88 Gallons.

And for ſuch Conical Tubs (or Tuns) whoſe Diameters are above 36 Inches, if they be near a Cylinder, one mean Inch will be ſufficient.

But for ſuch as are much Conical and great, you may find the whole Content by Problem V. Sect. VII. And if you deſire to Inch any ſuch Tun, obſerve the Directions in Problem I. Sect. VIII.

There are yet another ſort of Veſſels which Victuallers do frequently uſe to work their Ale in, and theſe are part of a Butt or Pipe ſet upon one Head, the other being cut off. The

The whole Content of theſe may be found by Problem VII. Section IX.

I have obſerved in ſeveral Places they cool their Worts in Braſs-Pans, whoſe Diameter is about 30 Inches, and Depth 10 or 12 Inches; but theſe are commonly deeper in the middle by an Inch or two, than at the Side.

In ſuch Veſſels as theſe, if you take (with the Rule) the Area of the Liquor's Surface, and multiply it by the Depth taken within 3 or 4 Inches of the Side, the Product will be the Content very near the truth.

And for round Bowls, which are like the Segment of a Globe, if you multiply the Area of the Liquor's Surface by half the Depth, it gives the Content near enough in Practice: But in ſuch Caſes as theſe, Experience will be your beſt Tutor.

But that I may not be wanting to the meaneſt Capacities, I have here inſerted a very uſeful Table, ſhewing the Area's of Circles, and Contents of Cylinders in Ale-Gallons.

The

THE
Explanation and Uſe
OF THE
TABLE *of Cylinders.*

THE Table conſiſts of two Parts: The firſt ſhews the *Area's* of *Circles*, and Contents of *Cylinders* from 12 to 42 Inches Diameter, and from 1 to 10 Inches deep, (and by Addition, to any other Depth whatſoever.) The ſecond Part ſhews only the *Area's* of *Circles* from 42 to 156 Inches Diameter.

In the firſt Part of the Table, the two firſt Pages, (and ſo every two next ſucceeding) contain 10 Columns: In the firſt of which you have the Diameters of Circles beginning at 12, and continued down (in the ſaid firſt Column on every left-hand Page) in Inches and Tenths, to 42 Inches.

The other Nine Columns are number'd at the Top, 1, 2, 3, 4, &c. to 9. In the firſt of theſe under 1, you have (againſt their reſpective Diameters) the *Area's* of *Circles* (or the Content at 1 Inch deep) to the hundred thouſandth part of an Ale-Gallon.

In

In the next Column under 2, is the Content at 2 Inches deep; under 3, the Content at 3 Inches deep; and ſo on to 9 Inches deep in the laſt Column under 9.

Thus if the Diameter of a Circle be 29.6 Inches, the Area is 2.4401 Gallons, the Content at 1 Inch deep. And if the Diameter of a *Cylinder* be 29.6 Inches,

The Content at	Inches Deep is	Gallons.
2		4.88
3		7.32
4		9.76
5		12.20
6		14.64
7		17.08
8		19.52
9		21.96

Note, If the former Cylinder of 29.6 Inches Diameter,

Be	Inches Deep, the Content will be	Gallons.
10		24.4
20		48.8
30		73.2
40		97.6
50		122.0
60		146.4
70		170.8
80		195.2
90		219.6

For the Content of any Cylinder at 2 Inches, and 20 Inches deep, is expreſs'd by the ſame Figures; but as 20 is ten times 2, the Area at 2 Inches deep muſt be made ten times greater, which is done by only putting the Prick, or Point one

one place more towards the Right-hand. From hence the Uſe of this Table is obvious, as may further appear by the following Example.

There is a Tun in the form of a Cylinder the Diameter is 29.6 *and the Depth* 27.8 *Inches, what is the Content in Gallons?*

At 29.6 Inches Diameter,

20	Inches deep is	48.80
7	Inches deep is	17.08
.8	of an Inch deep is	1.95
27.8	Content ſought is	67.83

If the mean Diameter of your Tun be above 42 Inches, find it in the firſt Column of the ſecond part of the Table, for againſt the whole Inches of your Diameter in the ſide of the Table, and under the tenths (if any be) you have the Area or Content at one Inch deep; this multiply'd by the whole Depth, gives the Content of the Tun; but this is ſo plain it needs no Example.

And for Casks, find the mean Diameter by the Direction given in Page 112, and then by the Length and mean Diameter the Content is found as above.

So if the Length of a Cask be 30 Inches, and the mean Diameter 23 Inches, the Content will be found to be 44.2 Gallons, for againſt 23 the Mean Diameter, and under 3 is 4.42. Now if 3 be 30, 4.42 will be 44.2, the Content ſought.

A

TABLE

OF

CYLINDERS,

From 12, to 42 Inches Diameter, and from 1, to 9 Inches Deep:

Very useful in Gauging of

Small Tuns and Casks;

SHEWING

By Inspection, the *Content* of such Vessels to the Thousandth Part of a Gallon, Ale-Measure.

Diam. in inches.	DEPTH.			
	1	2	3	4
12.0	0.4011	0.80	1.20	1.60
.1	0.4078	0.81	1.22	1.63
.2	0.4146	0.83	1.24	1.65
.3	0.4214	0.84	1.26	1.69
.4	0.4282	0.86	1.28	1.71
.5	0.4352	0.87	1.30	1.74
.6	0.4422	0.88	1.33	1.77
.7	0.4492	0.90	1.35	1.80
.8	0.4563	0.91	1.37	1.82
.9	0.4635	0.93	1.39	1.85
13.0	0.4707	0.94	1.41	1.88
.1	0.4780	0.96	1.43	1.91
.2	0.4853	0.97	1.46	1.94
.3	0.4927	0.98	1.46	1.97
.4	0.5001	1.00	1.50	2.00
.5	0.5076	1.01	1.52	2.03
.6	0.5151	1.03	1.55	2.06
.7	0.5227	1.04	1.57	2.09
.8	0.5304	1.06	1.59	2.12
.9	0.5381	1.08	1.61	2.15
14.0	0.5459	1.09	1.64	2.18
.1	0.5537	1.11	1.66	2.21
.2	0.5616	1.12	1.68	2.24
.3	0.5695	1.14	1.71	2.28
.4	0.5775	1.15	1.73	2.31
.5	0.5856	1.17	1.76	2.34
.6	0.5937	1.19	1.78	2.37
.7	0.6018	1.20	1.80	2.40
.8	0.6100	1.22	1.83	2.44
.9	0.6183	1.24	1.85	2.47

DEPTH				
5	6	7	8	9
2.01	2.41	2.81	3.21	3.61
2.04	2.44	2.85	3.26	3.67
2.07	2.49	2.90	3.31	3.73
2.11	2.53	2.95	3.37	3.79
2.14	2.57	3.00	3.43	3.86
2.18	2.61	3.05	3.48	3.92
2.21	2.65	3.10	3.54	3.98
2.24	2.69	3.14	3.59	4.04
2.28	2.73	3.19	3.65	4.10
2.32	2.78	3.24	3.71	4.17
2.35	2.82	3.30	3.77	4.24
2.39	2.87	3.35	3.82	4.30
2.43	2.91	3.39	3.88	4.36
2.46	2.95	3.44	3.94	4.43
2.50	3.00	3.50	4.00	4.50
2.53	3.04	3.55	4.06	4.56
2.58	3.09	3.61	4.12	4.63
2.61	3.13	3.65	4.18	4.70
2.65	3.18	3.71	4.24	4.77
2.69	3.23	3.77	4.30	4.84
2.73	3.27	3.82	4.36	4.91
2.76	3.32	3.87	4.42	4.98
2.81	3.37	3.93	4.49	5.05
2.84	3.41	3.98	4.55	5.12
2.88	3.46	4.04	4.62	5.19
2.93	3.51	4.10	4.68	5.26
2.96	3.56	4.15	4.74	5.34
3.01	3.61	4.21	4.81	5.41
3.05	3.66	4.27	4.88	5.49
3.09	3.71	4.33	4.94	5.56

Diam. in Inches.	DEPTH.			
	1	2	3	4
15.0	0.6266	1.25	1.88	2.50
.1	0.6350	1.27	1.91	2.54
.2	0.6435	1.29	1.93	2.57
.3	0.6520	1.30	1.96	2.61
.4	0.6604	1.32	1.98	2.64
.5	0.6691	1.34	2.01	2.68
.6	0.6778	1.35	2.03	2.71
.7	0.6865	1.37	2.06	2.74
.8	0.6953	1.39	2.09	2.78
.9	0.7041	1.41	2.11	2.82
16.0	0.7130	1.43	2.14	2.85
.1	0.7219	1.44	2.16	2.89
.2	0.7309	1.46	2.19	2.92
.3	0.7400	1.48	2.22	2.96
.4	0.7491	1.50	2.25	3.00
.5	0.7582	1.52	2.27	3.03
.6	0.7654	1.53	2.30	3.07
.7	0.7767	1.55	2.33	3.10
.8	0.7861	1.57	2.36	3.14
.9	0.7954	1.59	2.39	3.18
17.0	0.8049	1.61	2.41	3.22
.1	0.8144	1.63	2.44	3.26
.2	0.8239	1.64	2.47	3.29
.3	0.8335	1.67	2.50	3.33
.4	0.8432	1.69	2.53	3.37
.5	0.8529	1.70	2.56	3.41
.6	0.8627	1.72	2.58	3.45
.7	0.8725	1.74	2.62	3.49
.8	0.8824	1.76	2.65	3.53
.9	0.8924	1.78	2.68	3.57

DEPTH.				
5	6	7	8	9
3.13	3.76	4.38	5.01	5.63
3.18	3.81	4.45	5.08	5.72
3.22	3.86	4.50	5.15	5.79
3.26	3.91	4.56	5.22	5.87
3.30	3.96	4.62	5.28	5.94
3.35	4.01	4.68	5.35	6.02
3.39	4.06	4.74	5.42	6.09
3.43	4.12	4.80	5.49	6.18
3.48	4.17	4.87	5.56	6.26
3.52	4.22	4.93	5.63	6.34
3.57	4.28	4.99	5.70	6.42
3.61	4.33	5.05	5.78	6.50
3.65	4.39	4.12	5.85	6.58
3.70	4.44	5.18	5.92	6.66
3.75	4.49	5.24	5.99	6.74
3.79	4.55	5.31	6.06	6.82
3.84	4.60	5.37	6.14	6.90
3.88	4.66	5.43	6.21	6.99
3.93	4.71	5.50	6.28	7.07
3.98	4.77	5.57	6.36	7.16
4.02	4.83	5.63	6.44	7.24
4.07	4.88	5.70	6.51	7.33
4.12	4.94	5.77	6.59	7.42
4.17	5.00	5.83	6.66	7.50
4.22	5.06	5.90	6.74	7.59
4.26	5.11	5.97	6.82	7.68
4.31	5.17	6.03	6.90	7.76
4.36	5.23	6.10	6.98	7.85
4.41	5.29	6.17	7.06	7.94
4.46	5.35	6.24	7.14	8.03

Diam. in Inches.	DEPTH.			
	1	2	3	4
18.0	0.9024	1.80	2.71	3.61
.1	0.9124	1.82	2.74	3.65
.2	0.9225	1.84	2.77	3.69
.3	0.9327	1.87	2.80	3.73
.4	0.9429	1.88	2.83	3.77
.5	0.9532	1.91	2.86	3.81
.6	0.9635	1.93	2.89	3.85
.7	0.9739	1.95	2.92	3.90
.8	0.9843	1.97	2.95	3.94
.9	0.9948	1.99	2.98	3.98
19.0	1.0054	2.01	3.02	4.02
.1	1.0160	2.03	3.05	4.06
.2	1.0267	2.05	3.08	4.11
.3	1.0374	2.07	3.11	4.15
.4	1.0482	2.10	3.14	4.19
.5	1.0590	2.12	3.18	4.24
.6	1.0699	2.14	3.21	4.28
.7	1.0808	2.16	3.24	4.32
.8	1.0918	2.18	3.27	4.37
.9	1.1020	2.21	3.31	4.41
20.0	1.1140	2.23	3.34	4.46
.1	1.1252	2.25	3.38	4.50
.2	1.1364	2.27	3.41	4.55
.3	1.1477	2.29	3.44	4.59
.4	1.1590	2.32	3.48	4.64
.5	1.1704	2.34	3.51	4.68
.6	1.1819	2.36	3.54	4.73
.7	1.1934	2.39	3.58	4.77
.8	1.2049	2.41	3.61	4.82
.9	1.2165	2.43	3.65	4.86

DEPTH.				
5	6	7	8	9
4.51	5.41	6.31	7.22	8.12
4.56	5.47	6.38	7.30	8.21
4.61	5.53	6.45	7.38	8.30
4.66	5.59	6.53	7.46	8.39
4.71	5.66	6.60	7.54	8.49
4.77	5.72	6.67	7.63	8.58
4.82	5.78	6.74	7.71	8.67
4.87	5.85	6.82	7.79	8.77
4.92	5.90	6.89	7.87	8.86
4.97	5.97	6.96	7.95	8.95
5.03	6.03	7.04	8.04	9.05
5.08	6.10	7.11	8.13	9.14
5.13	6.16	7.18	8.21	9.23
5.19	6.22	7.26	8.30	9.34
5.24	6.29	7.34	8.38	9.43
5.30	6.35	7.41	8.47	9.53
5.35	6.42	7.49	8.56	9.63
5.40	6.49	7.57	8.65	9.73
5.46	6.55	7.64	8.74	9.83
5.51	6.62	7.72	8.82	9.93
5.57	6.68	7.80	8.91	10.03
5.63	6.75	7.88	9.00	10.13
5.68	6.82	7.95	9.09	10.23
5.74	6.88	8.03	9.18	10.33
5.80	6.95	8.11	9.27	10.43
5.85	7.02	8.19	9.36	10.53
5.91	7.09	8.27	9.46	10.64
5.97	7.16	8.35	9.54	10.74
6.02	7.23	8.43	9.64	10.84
6.08	7.30	8.51	9.73	10.94

Diam. in Inches.	DEPTH. 1	2	3	4
21.0	1.2282	2.46	3.68	4.91
.1	1.2400	2.48	3.72	4.96
.2	1.2517	2.50	3.75	5.01
.3	1.2635	2.53	3.79	5.05
.4	1.2754	2.55	3.83	5.10
.5	1.2874	2.57	3.86	5.15
.6	1.2994	2.60	3.90	5.20
.7	1.3114	2.62	3.93	5.25
.8	1.3236	2.65	3.97	5.29
.9	1.3357	2.67	4.01	5.34
22.0	1.3480	2.70	4.04	5.39
.1	1.3602	2.72	4.08	5.44
.2	1.3726	2.75	4.12	5.49
.3	1.3850	2.77	4.16	5.54
.4	1.3974	2.79	4.01	5.59
.5	1.4099	2.82	4.23	5.64
.6	1.4225	2.84	4.27	5.69
.7	1.4351	2.87	4.31	5.74
.8	1.4478	2.90	4.34	5.79
.9	1.4605	2.92	4.38	5.84
23.0	1.4733	2.95	4.42	5.89
.1	1.4861	2.97	4.46	5.94
.2	1.4990	3.00	4.50	6.01
.3	1.5120	3.02	4.54	6.05
.4	1.5250	3.05	4.58	6.10
.5	1.5380	3.08	4.61	6.15
.6	1.5511	3.10	4.65	6.20
.7	1.5643	3.13	4.69	6.26
.8	1.5775	3.15	4.73	6.31
.9	1.5908	3.18	4.77	6.36

DEPTH				
5	6	7	8	9
6.14	7.37	8.60	9.82	11.05
6.20	7.44	8.68	9.92	11.16
6.26	7.51	8.76	10.01	11.26
6.32	7.58	8.84	10.11	11.37
6.38	7.65	8.93	10.20	11.48
6.44	7.72	9.01	10.30	11.59
6.50	7.80	9.09	10.39	11.69
6.56	7.87	9.18	10.49	11.80
6.62	7.94	9.26	10.59	11.91
6.68	8.01	9.35	10.68	12.02
6.74	8.09	9.44	10.78	12.13
6.80	8.16	9.52	10.88	12.24
6.86	8.23	9.61	10.98	12.35
6.93	8.31	9.70	11.08	12.47
6.99	8.38	9.78	11.18	12.58
7.05	8.46	9.87	11.28	12.69
7.11	8.53	9.96	11.38	12.80
7.18	8.61	10.05	11.48	12.92
7.24	8.68	10.13	11.58	13.02
7.30	8.76	10.22	11.68	13.14
7.37	8.84	10.31	11.78	13.26
7.43	8.92	10.40	11.89	13.37
7.50	8.99	10.49	11.99	13.49
7.56	9.07	10.58	12.10	13.61
7.63	9.15	10.68	12.20	13.73
7.69	9.23	10.77	12.30	13.84
7.76	9.31	10.86	12.41	13.96
7.82	9.39	10.95	12.51	14.08
7.89	9.46	11.04	12.62	14.23
7.95	9.54	11.13	12.72	14.31

Diam. in Inches.	DEPTH.			
	1	2	3	4
24.0	1.6042	3.21	4.81	6.42
.1	1 6176	3.23	4.85	6.47
.2	1.6310	3.26	4.89	6.52
.3	1.6445	3.29	4.93	6.58
.4	1.6581	3.32	4.97	6 63
.5	1.6717	3.34	5.01	6.69
.6	1.6854	3.37	5.06	6.74
.7	1.6991	3.40	5.10	6.80
.8	1.7129	3.43	5.14	6.85
.9	1.7267	3.45	5.18	6.91
25.0	1.7406	3.48	5.22	6.96
.1	1.7546	3.51	5.26	7.02
.2	1.7686	3.54	5.31	7.07
.3	1.7827	3.57	5.35	7.13
.4	1.7968	3.59	5.39	7.19
.5	1.8110	3.62	5.43	7.24
.6	1.8252	3.65	5.48	7.30
.7	1.8395	3.68	5.52	7.36
.8	1.8538	3.71	5.56	7.42
.9	1.8682	3.74	5.60	7.47
26.0	1.8827	3.77	5.65	7.53
.1	1.8972	3.79	5.69	7.59
.2	1.9117	3.82	5.73	7.65
.3	1.9264	3.85	5.78	7.71
.4	1.9411	3.88	5.82	7.76
.5	1.9558	3.91	5.87	7.82
.6	1.9706	3.94	5.91	7.88
.7	1.9854	3.97	5.96	7.94
.8	2.0003	4.00	6.00	8.00
.9	2.0153	4.03	6.05	8.06

DEPTH.				
5	6	7	8	9
8.02	9.62	11.23	12.83	14.44
8.09	9.70	11.32	12.93	14.55
8.16	9.79	11.42	13.05	14.63
8.22	9.86	11.51	13.15	14.80
8.29	9.95	11.61	13.26	14.92
8.36	10.03	11.70	13.37	15 04
8.43	10.11	11.80	13.48	15.17
8.50	10.19	11.89	13.59	15.29
8.56	10.28	11.99	13.70	15.42
8.63	10.36	12.08	13.81	15.54
8.70	10.44	12.18	13.92	15.67
8.77	10.53	12.28	14.03	15.79
8.84	10.41	12.38	14.15	15.91
8.91	10.70	12.47	14.26	16.04
8.98	10.78	12.57	14.37	16.17
9.06	10.87	12.68	14.49	16.30
9.13	10.95	12.78	14.60	16.43
9.20	11.04	12.88	14.71	16.55
9.27	11.12	12.98	14.83	16.64
9.34	11.21	13.08	14.94	16.81
9.41	11.29	13.18	15.06	16.94
9.49	11.38	13.28	15.18	17.07
9.56	11.47	13.38	15.29	17.20
9.63	11.56	13.48	15.41	17.34
9.70	11.65	13.59	15.53	17.47
9.78	11.73	13.69	15.65	17.60
9.85	11.82	13.79	15.76	17.74
9.93	11.91	13.90	15.88	17.87
10.00	12.00	14.00	16.00	18.00
10.08	12.09	14.11	16.12	18.14

Diam. in Inches.	DEPTH.			
	1	2	3	4
27.0	2.0303	4.06	6.09	8.12
.1	2.0454	4.09	6.14	8.18
.2	2.0605	4.12	6.18	8.24
.3	2.0757	4.15	6.23	8.30
.4	2.0909	4.18	6.27	8.36
.5	2.1062	4.21	6.32	8.42
.6	2.1215	4.24	6.36	8.48
.7	2.1369	4.27	6.41	8.55
.8	2.1524	4.30	6.46	8.61
.9	2.1679	4.33	6.50	8.67
28.0	2.1835	4.37	6.55	8.73
.1	2.1991	4.40	6.60	8.80
.2	2.2148	4.43	6.64	8.86
.3	2.2305	4.46	6.69	8.92
.4	2.2463	4.49	6.74	8.98
.5	2.2621	4.52	6.79	9.05
.6	2.2780	4.56	6.83	9.11
.7	2.2940	4.59	6.88	9.18
.8	2.3100	4.62	6.93	9.24
.9	2.3261	4.65	6.98	9.30
29.0	2.3422	4.68	7.03	9.37
.1	2.3584	4.72	7.07	9.43
.2	2.3746	4.75	7.12	9.50
.3	2.3910	4.78	7.17	9.56
.4	2.4073	4.81	7.22	9.63
.5	2.4237	4.85	7.27	9.69
.6	2.4401	4.88	7.32	9.76
.7	2.4567	4.91	7.37	9.83
.8	2.4732	4.94	7.42	9.89
.9	2.4899	4.98	7.47	9.96

DEPTH.				
5	6	7	8	9
10.15	12.18	14.21	16.24	18.27
10.23	12.27	14.32	16.36	18.41
10.30	12.36	14.42	16.48	18.54
10.38	12.45	14.53	16.61	18.68
10.45	12.55	14.64	16.73	18.82
10.53	12.64	14.74	16.85	18.95
10.61	12.73	14.85	16.97	19.09
10.68	12.82	14.96	17.10	19.23
10.76	12.91	15.06	17.22	19.37
10.84	13.01	15.18	17.34	19.51
10.92	13.10	15.28	17.46	19.65
11.00	13.19	15.39	17.59	19.79
11.08	13.29	15.50	17.71	19.93
11.15	13.38	15.61	17.84	20.07
11.23	13.48	15.72	17.97	20.22
11.31	13.57	15.83	18.10	20.36
11.39	13.67	15.95	18.22	20.50
11.47	13.76	16.06	18.35	20.65
11.55	13.86	16.17	18.48	20.79
11.63	13.96	16.28	18.61	20.93
11.71	14.05	16.39	18.74	21.08
11.79	14.15	16.51	18.86	21.22
11.87	14.25	16.62	18.99	21.37
11.96	14.35	16.74	19.13	21.52
12.04	14.44	16.85	19.26	21.66
12.12	14.54	16.96	19.39	21.81
12.20	14.64	17.08	19.52	21.96
12.28	14.74	17.19	19.65	22.10
12.37	14.84	17.31	19.78	22.26
12.45	14.94	17.43	19.92	22.41

Diam. in Inches.	DEPTH.			
	1	2	3	4
30.0	2.5066	5.01	7.52	10.03
.1	2.5233	5.05	7.57	10.09
.2	2.5401	5.08	7.62	10.16
.3	2.5570	5.11	7.67	10.23
.4	2.5739	5.15	7.72	10.29
.5	2.5908	5.18	7.77	10.36
.6	2.6078	5.21	7.82	10.43
.7	2.6249	5.25	7.87	10.50
.8	2.6420	5.28	7.93	10.57
.9	2.6592	5.32	7.98	10.64
31.0	2.6764	5.35	8.03	10.71
.1	2.6938	5.39	8.08	10.78
.2	2.7111	5.42	8.13	10.84
.3	2.7285	5.46	8.19	10.91
.4	2.7460	5.49	8.24	10.98
.5	2.7635	5.53	8.29	11.05
.6	2.7811	5.56	8.34	11.12
.7	2.7987	5.60	8.40	11.19
.8	2.8163	5.63	8.45	11.27
.9	2.8341	5.67	8.50	11.34
32.0	2.8519	5.70	8.56	11.41
.1	2.8698	5.74	8.61	11.48
.2	2.8877	5.78	8.66	11.55
.3	2.9057	5.81	8.72	11.62
.4	2.9237	5.85	8.77	11.69
.5	2.9418	5.88	8.83	11.77
.6	2.9599	5.92	8.88	11.84
.7	2.9781	5.96	8.93	11.91
.8	2.9963	5.99	8.99	11.98
.9	3.0148	6.03	9.04	12.06

DEPTH.				
5	6	7	8	9
12.53	15.04	17.54	20.05	22.56
12.62	15.14	17.66	20.18	22.71
12.70	15.24	17.78	20.32	22.86
12.79	15.34	17.90	20.46	23.01
12.87	15.44	18.02	20.59	23.17
12.95	15.55	18.14	20.73	23.32
13.04	15.65	18.25	20.86	23.47
13.12	15.75	18.37	21.00	23.62
13.21	15.85	18.49	21.14	23.78
13.30	15.95	18.61	21.27	23.93
13.38	16.06	18.73	21.41	24.09
13.47	16.16	18.86	21.55	24.25
13.56	16.27	18.98	21.69	24.40
13.64	16.37	19.10	21.83	24.56
13.73	16.48	19.22	21.97	24.71
13.82	16.58	19.34	22.10	24.87
13.91	16.69	19.47	22.25	25.03
13.99	16.79	19.59	22.38	25.18
14.08	16.90	19.71	22.53	25.35
14.17	17.00	19.84	22.67	25.51
14.26	17.11	19.96	22.82	25.67
14.35	17.22	20.09	22.96	25.83
14.44	17.32	20.21	23.10	25.98
14.53	17.43	20.34	23.24	26.15
14.62	17.54	20.46	23.39	26.31
14.71	17.65	20.59	23.53	26.48
14.80	17.76	20.72	23.68	26.64
14.89	17.87	20.85	23.82	26.80
14.98	17.98	20.97	23.97	26.97
15.07	18.09	21.10	24.11	27.13

Diam. in Inches.	DEPTH.			
	1	2	3	4
33.0	3.0330	6.07	9.10	12.13
.1	3.0514	6.10	9.15	12.21
.2	3.0698	6.14	9.21	12.28
.3	3.0884	6.18	9.26	12.35
.4	3.1069	6.21	9.32	12.43
.5	3.1256	6.25	9.38	12.50
.6	3.1443	6.29	9.43	12.58
.7	3.1630	6.33	9.49	12.65
.8	3.1818	6.36	9.55	12.73
.9	3.2007	6.40	9.60	12.80
34.0	3.2196	6.44	9.66	12.88
.1	3.2385	6.48	9.71	12.95
.2	3.2576	6.51	9.77	13.03
.3	3.2766	6.55	9.83	13.11
.4	3.2958	6.59	9.89	13 18
.5	3.3150	6.63	9.95	13.26
.6	3.3342	6.67	10.00	13.34
.7	3.3535	6.71	10.06	13.41
.8	3.3728	6.75	10.12	13.49
.9	3.3923	6.78	10.18	13.57
35.0	3.4117	6.82	10.24	13.65
.1	3.4312	6.86	10.29	13.73
.2	3.4508	6.90	10.35	13.80
.3	3.4705	6.94	10.41	13.88
.4	3.4902	6.98	10.47	13.96
.5	3.5099	7 02	10.53	14.04
.6	3.5297	7.06	10.59	14.12
.7	3.5496	7.10	10.65	14.20
.8	3.5605	7.14	10.71	14.28
.9	3.5894	7.18	10.77	14.36

DEPTH				
5	6	7	8	9
15.17	18.20	21.23	24.26	27.30
15.26	18.31	21.36	24.41	27.46
15.35	18.42	21.49	24.56	27.63
15.44	18.53	21.62	24.71	27.79
15.53	18.64	21.75	24.85	27.96
15.63	18.75	21.88	25.00	28.13
15.72	18.86	22.01	25.15	28.30
15.82	18.98	22.14	25.30	28.47
15.91	19.09	22.27	25.45	28.63
16.00	19.20	22.40	25.60	28.80
16.10	19.32	22.54	25.75	28.97
16.19	19.43	22.67	25.90	29.14
16.29	19.54	22.80	26.06	29.32
16.38	19.66	22.93	26.21	29.49
16.48	19.77	22.07	26.36	29.66
16.58	19.89	23.21	26.52	29.84
16.67	20.00	23.34	26.67	30.01
16.77	20.12	23.47	26.82	30.18
16.86	20.23	23.61	26.98	30.35
16.96	20.35	23.75	26.14	30.53
17.06	20.47	23.88	27.29	30.70
17.16	20.59	24.01	27.45	30.88
17.25	20.70	24.16	27.61	31.06
17.35	20.82	24.29	27.76	31.24
17.45	20.94	24.43	27.92	31.41
17.55	21.06	24.57	28.08	31.59
17.65	21.18	24.71	28.24	31.76
17.75	21.30	24.85	28.39	31.94
17.85	21.42	24.99	28.55	32.13
17.95	21.53	25.12	28.71	32.30

Diam. in Inches.	DEPTH.			
	1	2	3	4
36.0	3.6065	7.22	10.83	14.44
.1	3.6295	7.26	10.89	14.52
.2	3.6497	7.30	10.95	14.60
.3	3.6699	7.34	11.01	14.68
.4	3.6901	7.38	11.07	14.76
.5	3.7104	7.42	11.13	14.84
.6	3.7308	7.46	11.19	14.92
.7	3.7512	7.50	11.25	15.00
.8	3.7717	7.54	11.32	15.09
.9	3.7922	7.58	11.38	15.17
37.0	3.8128	7.63	11.44	15.25
.1	3.8334	7.67	11.50	15.33
.2	3.8541	7.71	11.56	15.42
.3	3.8749	7.75	11.62	15.50
.4	3.8957	7.79	11.69	15.58
.5	3.9165	7.83	11.75	15.67
.6	3.9374	7.87	11.81	15.75
.7	3.9584	7.92	11.88	15.83
.8	3.9794	7.96	11.94	15.92
.9	4.0005	8.00	12.00	16.00
38.0	4.0217	8.04	12.06	16.09
.1	4.0428	8.09	12.13	16.17
.2	4.0641	8.13	12.17	16.24
.3	4.0854	8.17	12.26	16.34
.4	4.1068	8.21	12.32	16.43
.5	4.1282	8.26	12.38	16.51
.6	4.1496	8.30	12.45	16.60
.7	4.1712	8.34	12.51	16.68
.8	4.1927	8.39	12.58	16.77
.9	4.2144	8.43	12.64	16.86

DEPTH.				
5	6	7	8	9
18.05	21.66	25.27	29.87	32.48
18.15	21 78	25.41	29.03	32.66
18.25	21.90	25.55	29.19	32.84
18.35	22.02	25.69	29.36	33.03
18.45	22.14	25.83	29.52	33.21
18.55	22.26	25.97	29.68	33.39
18 65	22.39	26.11	29.85	33.58
18.76	22.51	26.26	30.01	33.76
18.86	22.63	26.40	30.17	33.94
18.96	22.75	26.55	30.34	34.13
19.07	22.88	26.69	30.51	34.32
19.17	23.00	26.83	30.67	34.50
19.27	23.12	26.98	30.83	34.69
19.37	23.25	27.12	31.00	34.87
19.48	23.37	27.27	31.17	35.06
19.58	23.50	27.41	31.33	35.25
19.69	23.62	27.56	31.50	35.43
19.79	23.75	27.71	31.66	35.62
19.90	23.87	27.86	31.83	35.81
20.00	24.00	28.00	32.00	36.00
20.11	24.13	28.15	32.17	36.19
20.21	24.26	28.30	32.34	36.39
20.30	24.38	28.44	32.49	36.56
20.43	24.51	28.60	32.68	36.77
20.54	24.64	28.75	32.85	36.96
20.64	24.77	28.90	33.02	37.15
20.75	24.90	29.05	33.19	37.34
20.86	25.03	29.20	33.37	37.54
20.96	25.15	29.35	33.54	37.73
21.07	25.29	29.50	33.71	37.93

Diam. in Inches.	DEPTH.			
	1	2	3	4
39.0	4.2361	8.47	12.71	16.94
.1	4.2578	8.52	12.77	17.03
.2	4.2796	8.56	12.84	17.12
.3	4.3015	8.60	12.90	17.20
.4	4.3234	8.65	12.97	17.29
.5	4.3454	8.69	13.04	17.38
.6	4.3674	8.73	13.10	17.47
.7	4.3895	8.78	13.17	17.56
.8	4.4117	8.82	13.24	17.65
.9	4.4338	8.87	13.30	17.74
40.0	4.4562	8.91	13.37	17.82
.1	4.4785	8.96	13.43	17.91
.2	4.5008	9.00	13.50	18.00
.3	4.5233	9.05	13.57	18.09
.4	4.5457	9.09	13.64	18.18
.5	4.5683	9.14	13.71	18.27
.6	4.5908	9.18	13.77	18.36
.7	4.6135	9.23	13.84	18.45
.8	4.6362	9.27	13.91	18.54
.9	4.6589	9.32	13.98	18.64
41.0	4.6818	9.36	14.05	18.73
.1	4.7046	9.41	14.11	18.82
.2	4.7275	9.45	14.18	18.91
.3	4.7505	9.50	14.25	19.00
.4	4.7735	9.55	14.32	19.09
.5	4.7966	9.59	14.39	19.19
.6	4.8198	9.64	14.46	19.28
.7	4.8430	9.69	14.53	19.37
.8	4.8662	9.73	14.60	19.46
.9	4.8895	9.78	14.67	19.56

DEPTH.				
5	6	7	8	9
21.18	25.42	29.65	33.89	38.12
21.29	25.55	29.81	34.06	38.32
21.40	25.68	29.95	34.23	38.51
21.51	25.81	30.11	34.41	38.71
21.62	25.94	30.26	34.58	38.91
21.73	26.07	30.42	34.76	39.11
21.84	26.20	30.57	34.94	39.30
21.95	26.33	30.72	35.11	39.50
22.06	26.47	30.89	35.30	39.71
22.17	26.60	31.04	35.47	39.91
22.28	26.74	31.19	35.65	40.11
22.39	26.87	31.35	35.83	40.30
22.50	27.00	31.51	36.01	40.51
22.62	27.14	31.66	36.18	40.71
22.73	27.28	31.82	36.37	40.93
22.84	27.41	31.98	36.54	41.11
22.95	27.55	32.14	36.73	41.32
23.07	27.68	32.29	36.91	41.52
23.18	27.82	32.45	37.09	41.72
23.30	27.95	32.61	37.27	41.93
23.41	28.09	32.77	37.46	42.14
23.52	28.22	32.93	37.63	42.34
23.64	28.36	33.09	37.82	42.54
23.75	28.50	33.25	38.00	42.75
23.87	28.64	33.41	38.19	42.96
23.98	28.78	33.57	38.37	43.17
24.10	28.92	33.74	38.56	43.38
24.22	29.06	33.90	38.74	43.59
24.33	29.19	34.06	38.93	43.80
24.45	29.34	34.23	39.11	44.00

THE TABLE

Continued,

From 42, to 156 Inches DIAMETER,

Shewing the

AREA'S

OF

CIRCLES,

TO THE

Ten Thouſandth part of a Gallon, Ale-Meaſure.

Dia.	.0	.1	.2	.3	.4
42	4.9129	4.9363	4.9598	4.9834	5.0069
43	5.1496	5.1736	5.1977	5.2218	5.2459
44	5.3920	5.4165	5.4411	5.4657	5.4904
45	5.6398	5.6649	5.6901	5.7153	5.7405
46	5.8933	5.9189	5.9446	5.9704	5.9962
47	6.1523	6.1735	6.2048	6.2311	6.2575
48	6.4169	6.4436	6.4705	6.4973	6.5243
49	6.6870	6.7143	6.7417	6.7692	6.7966
50	6.9628	6.9906	7.0186	7.0466	7.0746
51	7.2440	7.2725	7.3010	7.3296	7.3581
52	7.5309	7.5595	7.5882	7.6181	7.6472
53	7.8233	7.8529	7.8825	7.9122	7.9419
54	8.1214	8.1515	8.1816	8.2118	8.2421
55	8.4249	8.4556	8.4863	8.5171	8.5479
56	8.7341	8.7653	8.7966	8.8279	8.8593
57	9.0489	9.0806	9.1124	9.1443	9.1762
58	9.3691	9.4014	9.4338	9.4662	9.4988
59	9.6949	9.7278	9.7608	9.7938	9.8268
60	10.0264	10.0598	10.0933	10.1269	10.1605
61	10.3634	10.3974	10.4314	10.4655	10.4997
62	10.7059	10.7405	10.7751	10.8098	10.8445
63	11.0541	11.0892	11.1244	11.1596	11.1949
64	11.4078	11.4434	11.4792	11.5150	11.5598
65	11.7670	11.8033	11.8396	11.8759	11.9123
66	12.1319	12.1687	12.2055	12.2424	12.2794
67	12.5023	12.5357	12.5771	12.6145	12.6520
68	12.8723	12.9167	12.9542	12.9922	13.0305
69	13.2599	13.2985	13.3368	13.3754	13.4140
70	13.6476	13.6860	13.7251	13.7642	13.8034
71	14.0397	14.0793	14.1189	14.1586	14.1983

	.5	.6	.7	.8	.9
	5.0306	5.0543	5.0780	5.1019	5.1257
	5.2701	5.2944	5.3187	5.3430	5.3675
	5.5152	5.5400	5.5649	5.5898	5.6148
	5.7659	5.7912	5.8167	5.8425	5.8677
	6.0221	6.0480	6.0740	6.1000	6.1261
	6.2839	6.3104	6.3369	6.3635	6.3902
	6.5513	6.5783	6.6054	6.6325	6.6600
	6.8242	6.8518	6.8794	6.9072	6.9349
	7.1027	7.1309	7.1591	7.1873	7.2157
	7.3868	7.4155	7.4443	7.4731	7.5020
	7.6764	7.7057	7.7350	7.7644	7.7939
	7.9717	8.0015	8.0314	8.0613	8.0913
	8.2724	8.3028	8.3333	8.3638	8.3943
	8.5788	8.6097	8.6407	8.6718	8.7029
	8.8907	8.9222	8.9538	8.9854	9.0171
	9.2082	9.2403	9.2734	9.3046	9.3368
	9.5313	9.5635	9.5966	9.6293	9.6621
	9.8600	9.8931	9.9263	9.9596	9.9930
	10.1942	10.2275	10.2617	10.2955	10.3294
	10.5339	10.5682	10.6026	10.6370	10.6714
	10.8793	10.9141	10.9490	10.9840	11.0190
	11.2302	11.2656	11.3011	11.3366	11.3721
	11.5867	11.6227	11.6587	11.6947	11.7309
	11.9488	11.9853	12.0219	12.0585	12.0952
	12.3164	12.3535	12.3906	12.4278	12.4650
	12.6896	12.7272	12.7649	12.8027	12.8405
	13.0684	13.1066	13.1448	13.1831	13.2215
	13.4527	13.4915	13.4915	13.5691	13.6080
	13.8426	13.8819	13.8819	13.9607	14.0002
	14.2381	14.2780	14.2780	14.3579	14.3979

Dia.	.0	.1	.2	.3	.4
72	14.4380	14.4781	14.5183	14.5585	14.5988
73	14.8418	14.8825	14.9232	14.9640	15.0049
74	14.2512	15.2925	15.3338	15.3751	15.4165
75	15.8662	15.7080	15.7499	15.7918	15.8337
76	16.0867	16.1291	16.1715	16.2140	16.2565
77	16.5129	16.5558	16.5988	16.6418	16.6849
78	16.9445	16.9880	17.0316	17.0751	17.1188
79	17.3818	17.4258	17.4699	17.5141	17.5583
80	17.8246	17.8692	17.9139	17.9586	18.0033
81	18.2730	18.3182	18.3634	18.4086	18.4540
82	18.7270	18.7727	18.8185	18.8643	18.9102
83	19.1866	19.2328	19.2791	19.3255	19.3719
84	19.6517	19.6985	19.7454	19.7923	19.8393
85	20.1223	20.1697	20.2172	20.2646	20.3122
86	20.5986	20.6465	20.6945	20.7426	21.7907
87	21.0804	21.1289	21.1775	21.2261	21.2747
88	21.5678	21.6169	21.6660	21.7151	21.7643
89	22.0608	22.1104	22.1600	22.2098	22.2595
90	22.5593	22.6095	22.6597	22.7100	22.7603
91	23.0634	23.1141	23.1649	23.2157	23.2666
92	23.5731	23.6244	23.6757	23.7271	23.7785
93	24.0883	24.1402	24.1920	24.2440	24.2960
94	24.6091	24.6615	24.7140	24.7665	24.8190
95	25.1355	25.1885	25.2415	25.2945	25.2376
96	25.6675	25.7210	25.7745	25.8282	25.8818
97	26.2050	26.2591	26.3132	26.3673	26.4216
98	26.7481	26.8027	26.8574	26.9121	26.9669
99	27.2968	27.3519	27.4072	27.4625	27.5178
100	27.8510	27.9067	27.9625	28.0184	28.0743
101	28.4108	28.4671	28.5234	28.5798	28.6363

.5	.6	.7	.8	.9
14.6392	14.6796	14.7201	14.7606	14.8012
15.0458	15.0868	15.1278	15.1689	15.2100
15.4580	15.495C	15.5411	15.5827	15.6244
15.8758	15.9178	15.9600	16.0022	16.0444
16.2991	16.3417	16.3844	16.4272	16.4700
16.7280	16.7712	16.8145	16.8578	16.9011
17.1625	17.2062	17.2500	17.2939	17.3378
17.6025	17.6468	17.6912	17.7366	17.7801
18.0481	18.093C	18.1379	18.1829	18.2280
18.4993	18.5448	18.5902	18.6358	18.6814
18.9561	19.0021	19.0481	19.0942	19.1403
19.4184	19.4650	19.5115	19.5582	19.6049
19.8863	19.9334	19.9806	20.0278	20.0750
20.3598	20.4074	20.4551	20.5029	20.5507
20.8388	20.887C	20.9353	20.9836	21.0320
21.3234	21.3722	21.4210	21.4699	21.5188
21.8136	21.8629	21.9123	21.9617	22.0112
22.3093	22.3592	22.4092	22.4592	22.5092
22.8107	22.8611	22.9116	22.9621	23.0128
23.3176	23.3685	23.4196	23.4707	23.5220
23.8300	23.8816	23.9332	23.9848	24.0367
24.3480	24.4001	24.4523	24.5045	24.5568
24.8716	24.9243	24.9770	25.0298	25.0826
25.4008	25.4540	25.5073	25.5606	25.6140
25.9355	25.9893	26.0432	26.0671	26.1510
26.4759	26.5302	26.5846	26.6090	26.6935
27.0127	27.0766	27.1316	27.1866	27.2416
27.5732	27.6286	27.6841	27.7397	27.7953
28.1302	28.1862	28.2423	28.2984	28.3546
28.6928	28.7494	28.8060	28.8627	28.9194

Dia.	.0	.1	.2	.3	.4
102	28.9763	29.0330	29.0899	29.1469	29.2039
103	29.5471	29.6045	29.6620	29.2195	29.7771
104	30.1236	30.1816	30.2396	30.2977	30.3558
105	30.7057	30.7642	30.8228	30.8814	30.9401
106	31.2934	31.3525	31.4116	31.4708	31.5300
107	31.8866	31.9462	32.0059	32.0657	32.1255
108	32.4854	32.5456	32.6058	32.6661	32.7265
109	33.0898	33.1505	33.2113	33.2722	33.3331
110	33.6997	33.7610	33.8224	33.8838	33.9452
111	34.3152	34.3771	34.4390	34.5010	34.5630
112	34.9363	34.9987	35.0612	35.1237	35.1863
113	35.5629	35.6259	35.6889	35.7520	35.8152
114	36.1952	36.2587	36.3223	36.3859	36.4496
115	36.8329	36.8970	36.9612	37.0254	37.0896
116	37.4763	37.5409	37.6056	37.6704	37.7352
117	38.1252	38.1904	38.2557	38.3210	38.3864
118	38.7797	38.8455	38.9113	38.9772	39.0431
119	39.4398	39.5061	39.5725	39.6389	39.7054
120	40.1054	40.1723	40.2392	40.3062	40.3733
121	40.7766	40.8441	40.9116	40.9791	41.0467
122	41.4534	41.5214	41.5895	41.6575	41.7257
123	42.1358	42.2043	42.2729	42.3416	42.4103
124	42.8237	42.8928	42.9619	43.0312	43.1004
125	43.5172	43.5868	43.6566	43.7263	43.7961
126	44.2162	44.2865	44.3567	44.4271	44.4974
127	44.9209	44.9916	45.0625	45.1334	45.2043
128	45.6311	45.7024	45.7738	45.8452	45.9167
129	46.3468	46.4187	46.4907	46.5627	46.6347
130	47.0682	47.1406	47.2131	47.2857	47.3583
131	47.7951	47.8681	48.9412	48.0143	48.0874

	.5	.6	.7	.8	.9
	29.2610	29.3181	29.3753	29.4325	29.4898
	29.8347	29.8924	29.9501	30.0079	30.0657
	30.4140	30.4722	30.5305	30.5889	30.6473
	30.9989	31.0577	31.1165	31.1754	31.2344
	31.5893	31.6487	31.7081	31.7675	31.8270
	32.1853	32.2452	32.3052	32.3652	32.4252
	32.7869	32.8474	32.9079	32.9685	33.0291
	33.3940	33.4551	33.5161	33.5773	33.6385
	34.0068	34.0683	34.1300	34.1917	34.2534
	34.6251	34.6872	34.7494	34.8116	34.8739
	35.2489	35.3116	35.3744	35.4372	35.5000
	35.8784	35.9416	36.0049	36.0683	36.1317
	36.5134	36.5772	36.6410	36.7049	36.7689
	37.1539	37.2183	37.2827	37.3472	37.4117
	37.8001	37.8656	37.9300	37.9950	38.0601
	38.4518	38.5173	38.5828	38.6484	38.7140
	39.1091	39.1751	39.2412	39.3073	39.3735
	39.7719	39.8385	39.9052	39.9719	40.0386
	40.4403	40.5075	40.5747	40.6420	40.7093
	41.1143	41.1820	41.2498	41.3176	41.3855
	41.7939	41.8622	41.9305	41.9989	42.0673
	42.4790	42.5475	42.6167	42.6858	42.7547
	43.1697	43.2391	43.3086	43.3780	43.4476
	43.8660	43.9360	44.0059	44.0760	44.1461
	44.5679	44.6384	44.7089	44.7795	44.8502
	45.2753	45.3463	45.4174	45.4886	45.5598
	45.9883	46.0599	46.1315	46.2032	46.2750
	46.7068	46.7790	46.8512	46.9235	46.9958
	47.4309	47.5037	47.5764	47.6493	47.7222
	48.1606	48.2339	48.3073	48.3806	48.4541

Dia.	.0	.1	.2	.3	.4
132	48.5276	48.6011	48.6747	48.7484	48.8221
133	49.2656	49.3397	49.4139	49.4881	49.5624
134	50.0093	50.0839	50.1586	50.2334	50.3083
135	50.7584	50.8340	50.9099	50.9842	51.0600
136	51.5132	51.5890	51.6650	51.7410	51.8170
137	52.2740	52.3500	52.4262	52.5030	52.5792
138	53.0394	53.1164	53.1933	53.2703	53.3474
139	53.8109	53.8884	53.9659	54.0434	54.1211
140	54.5880	54.6660	54.7440	54.8222	54.9003
141	55.3706	55.4491	55.5268	55.6064	55.6852
142	56.1587	56.2379	56.3170	56.3963	56.4756
143	56.9525	57.0322	57.1119	57.1917	57.2716
144	57.7518	57.8321	57.9124	57.9927	58.0731
145	58.5567	58.6375	58.7184	58.7993	58.8802
146	59.3672	59.4485	59.5299	59.6114	59.6929
147	60.1832	60.2651	60.3471	60.4291	60.5112
148	61.0048	61.0873	61.1698	61.2524	61.3350
149	61.8320	61.9150	61.9981	62.0812	62.1644
150	62.6648	62.7483	62.8320	62.9157	62.9994
151	63.5030	63.5872	63.6714	63.7556	63.8400
152	64.3470	64.4316	64.5164	64.6012	64.6861
153	65.1964	65.2817	65.3670	65.4523	65.5377
154	66.0514	66.1372	69.2231	66 3090	66.3950
155	66.9120	66.9984	67.0848	67.1713	67.2578
156					

	.5	.6	.7	.8	.9
	48.8959	48.9697	49.0436	49.1176	49.1916
	49.6367	49.7111	49.7856	49.8601	49.9346
	50.3832	50.4581	50.5331	40.6082	50.6833
	51.1351	51.2110	51.2861	51.3620	51.4374
	51.8930	51.9690	52.0450	52.1210	52.1972
	52.6560	52.7324	52.8090	52.8860	52.9630
	53.4245	53.5017	53.5789	53.6562	53.7335
	54.1987	54.2765	54.3543	54.4321	54.5100
	54.9786	55.0569	55.1325	55.2136	55.2921
	55.7640	55.8428	55.9217	56.0007	56.0797
	56.5549	56.6343	56.7138	56.7933	56.8729
	57.3515	57.4314	57.5114	57.5915	57.6716
	58.1536	58.2341	58.3147	58.3953	58.4760
	58.9613	59.0423	59.1235	59.2047	59.2859
	59.7745	59.8561	59.9378	60.0196	60.1014
	60.5933	66.6755	60.7578	60.8401	60.9224
	61.4177	61.5005	61.5833	61.6661	61.7490
	62.2477	62.3310	62.4143	62.4978	62.5812
	63.0832	63.1671	63.2510	63.3350	63.4190
	63.9243	64.0087	64.0932	64.1777	64.2623
	64.7710	64.8560	64.9410	65.0261	65.1112
	65.6232	65.7088	65.7943	65.8800	65.9657
	66.4810	66.5671	66.6533	66.7395	66.8257
	67.3444	67.4311	67.5178	67.6045	67.6013

Circles

Dia	.0	.1	.2	.3	.4
1	0.0034	0.0041	0.0049	0.0057	0.0067
2	0.0136	0.0150	0.0165	0.0180	0.0196
3	0.0306	0.0327	0.0348	0.0370	0.0393
4	0.0544	0.0572	0.0600	0.0629	0.0658
5	0.0850	0.0884	0.0919	0.0955	0.0991
6	0.1224	0.1265	0.1307	0.1349	0.1393
7	0.1666	0.1714	0.1763	0.1812	0.1862
8	0.2176	0.2231	0.2286	0.2342	0.2399
9	0.2754	0.2816	0.2878	0.2941	0.3004
10	0.3400	0.3468	0.3537	0.3607	0.3677
11	0.4114	0.4189	0.4265	0.4341	0.4419
12	0.4896	0.4978	0.5061	0.5144	0.5228
13	0.5746	0.5835	0.5924	0.6014	0.6105
14	0.6664	0.6760	0.6856	0.6953	0.7050
15	0.7650	0.7752	0.7855	0.7959	0.8063
16	0.8704	0.8813	0.8923	0.9033	0.9145
17	0.9826	0.9942	1.0059	1.0176	1.0294
18	1.1016	1.1139	1.1262	1.1386	1.1511
19	1.2274	1.2404	1.2534	1.2665	1.2796
20	1.3600	1.3736	1.3873	1.4011	1.4149
21	1.4994	1.5137	1.5281	1.5425	1.5571
22	1.6456	1.6606	1.6757	1.6908	1.7060
23	1.7986	1.8143	1.8300	1.8458	1.8617
24	1.9584	1.9748	1.9912	2.0077	2.0242
25	2.1250	2.1420	2.1591	2.1763	2.1935
26	2.2984	2.3161	2.3339	2.3517	2.3697
27	2.4786	2.4970	2.5155	2.5340	2.5526
28	2.6656	2.6848	2.7038	2.7230	2.7423
29	2.8594	2.8792	2.8990	2.9189	2.9388
30	3.0600	3.0804	3.1009	3.1215	3.1421

	.5	.6	.7	.8	.9
	0.0077	0.0087	0.0098	0.0110	0.0123
	0.0213	0.0230	0.0248	0.0267	0.0286
	0.0417	0.0441	0.0465	0.0491	0.0517
	0.0689	0.0719	0.0751	0.0781	0.0816
	0.1029	0.1066	0.1105	0.1144	0.1184
	0.1437	0.1481	0.1526	0.1572	0.1619
	0.1913	0.1964	0.2016	0.2069	0.2122
	0.2457	0.2515	0.2573	0.2633	0.2693
	0.3069	0.3133	0.3199	0.3265	0.3332
	0.3748	0.3820	0.3893	0.3966	0.4040
	0.4496	0.4575	0.4654	0.4734	0.4815
	0.5312	0.5398	0.5484	0.5571	0.5658
	0.6196	0.6289	0.6381	0.6475	0.6569
	0.7148	0.7247	0.7347	0.7447	0.7548
	0.8168	0.8274	0.8381	0.8488	0.8596
	0.9256	0.9369	0.9442	0.9596	0.9711
	1.0412	1.0532	1.0652	1.0773	1.0894
	1.1636	1.1763	1.1889	1.2017	1.2145
	1.2928	1.3061	1.3195	1.3329	1.3464
	1.4288	1.4428	1.4569	1.4710	1.4852
	1.5716	1.5863	1.6010	1.6158	1.6307
	1.7212	1.7366	1.7520	1.7675	1.7830
	1.8776	1.8937	1.9097	1.9259	1.9421
	2.0408	2.0575	2.0743	2.0911	2.1080
	2.2108	2.2282	2.2457	2.2632	2.2808
	2.3876	2.4057	2.4238	2.4420	2.4603
	2.5712	2.5900	2.6088	2.6277	2.6466
	2.7616	2.7811	2.8005	2.8201	2.8396
	2.9588	2.9789	2.9991	3.0193	3.0396
	3.1628	3.1836	3.2045	3.2254	3.2464

Dia	.0	.1	.2	.3	.4
31	3.2674	3.2885	3.3097	3.3309	3.3523
32	3.4816	3.5034	3.5253	3.5472	3.5692
33	3.7026	3.7251	3.7476	3.7702	3.7929
34	3.9304	3.9536	3.9768	4.0001	4.0234
35	4.1650	4.1888	4.2127	4.2367	4.2607
36	4.4064	4.4309	4.4555	4.4801	4.5049
37	4.6546	4.6798	4.7051	4.7304	4.7558
38	4.9096	4.9355	4.9614	4.9874	5.0135
39	5.1714	5.1980	5.2246	5.2513	5.2780
40	5.4400	5.4672	5.4945	5.5219	5.5493
41	5.7154	5.7433	5.7713	5.7993	5.8275
42	5.9976	6.0262	6.0549	6.0836	6.1124
43	6.2866	6.3159	6.3452	6.3746	6.4041
44	6.5824	6.6124	6.6424	6.6725	6.7026
45	6.8850	6.9156	6.9463	6.9771	7.0079
46	7.1944	7.2257	7.2571	7.2885	7.3201
47	7.5106	7.5426	7.5747	7.6068	7.6390
48	7.8336	7.8663	7.8990	7.9318	7.9647
49	8.1634	8.1968	8.2302	8.2637	8.2972
50	8.5000	8.5340	8.5681	8.6023	8.6365
51	8.8434	8.8781	8.9129	8.9477	8.9827
52	9.1936	9.2290	9.2645	9.3000	9.3356
53	9.5506	9.5867	9.6228	9.6590	9.6953
54	9.9144	9.9512	9.6880	10.0249	10.0618
55	10.2850	10.3224	10.3599	10.3975	10.4351
56	10.6624	10.7005	10.7387	10.7769	10.8153
57	11.0466	11.0854	11.1243	11.1632	11.2022
58	11.4376	11.4771	11.5166	11.5562	11.5959
59	11.8354	11.8756	11.9158	11.9561	11.9964
60	12.2400	12.2808	12.3217	12.3627	12.4037

	.5	.6	.7	.8	.9
	3.3736	3.3951	3.4166	3.4382	3.4599
	3.5912	3.6134	3.6356	3.6579	3.6802
	3.8156	3.8385	3.8613	3.8843	3.9073
	4.0468	4.0703	4.0939	4.1175	4.1412
	4.2848	4.3090	4.3333	4.3576	4.3820
	4.5296	4.5545	4.5794	4.6044	4.6295
	4.7812	4.8068	4.8324	4.8581	4.8838
	5.0396	5.0659	5.0921	5.1185	5 1449
	5.3048	5.3317	5.3587	5.3857	5.4128
	5.5768	5.6044	5.6321	5.6598	5.6876
	5.8556	5.8839	5.9122	5.9406	5.9691
	6.1412	6.1702	6.1992	6.2283	6.2574
	6.4336	6.4633	6.4929	6.5227	6.5525
	6.7328	6.7631	6.7935	6.8239	6.8544
	7.0388	7.0698	7.1009	7.1320	7.1632
	7.3516	7.3833	7.4150	7.4468	7.4787
	7.6712	7.7036	7.7360	7.7685	7.8010
	7.9976	8.0307	8.0637	8.0969	8.1301
	8.3308	8.3645	8.3983	8.4321	8.4660
	8.6708	8.7052	8.7397	8.7742	8.8088
	9.0176	9.0527	9.0878	9.1230	9.1583
	9.3712	9.4070	9.4428	9.4787	9.5146
	9.7316	9.7681	9.8045	9.8411	9.8577
	10.0988	10.1359	10.1731	10.2103	10.2476
	10.4728	10.5106	10.5485	10.5864	10.6244
	10.8536	10.8921	10.9306	10.9692	11.0079
	11.2412	11.2804	11.3196	11.3589	11.3982
	11.6256	11.6755	11.7153	11.7553	11.7953
	12.0368	12.0773	12.1179	12.1585	12.1992
	11.4448	12.4860	12.5273	12.5686	12.6100

Excise for Common Brewers in the Country, at 4 *s.* 9 *d.* *per* Barrel, the Allowances being 2 ½ Barrels in every 23.

B.	0					¼					½					¾				
	l.	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23
1	0	4	2	3	5	0	5	3	2	0	0	6	4	0	19	0	7	4	3	14
2	0	8	5	2	10	0	9	6	1	5	0	10	7	0	1	0	11	7	2	19
3	0	12	8	1	15	0	13	9	0	10	0	14	9	3	6	0	15	10	2	1
4	0	16	11	0	20	0	17	11	3	15	0	19	0	2	11	1	0	1	1	6
5	1	1	2	0	2	1	2	2	2	20	1	3	3	1	16	1	4	4	0	11
6	1	5	4	3	7	1	6	5	2	2	1	7	6	0	21	1	8	6	3	16
7	1	9	7	2	12	1	10	8	1	7	1	11	9	0	3	1	12	9	2	21
8	1	13	10	1	17	1	14	11	0	12	1	15	11	3	8	1	17	0	2	3
9	1	18	1	0	22	1	19	1	3	17	2	0	2	2	13	2	1	3	1	8
10	2	2	4	0	4	2	3	4	2	22	2	4	5	1	18	2	5	6	0	13
11	2	6	6	3	9	2	7	7	2	4	2	8	8	1	0	2	9	8	3	18
12	2	10	9	2	14	2	11	10	1	9	2	12	11	0	5	2	13	11	3	0
13	2	15	0	1	19	2	16	1	0	14	2	17	1	3	10	2	18	2	2	5
14	3	19	3	1	1	3	0	3	3	19	3	1	4	2	15	3	2	5	1	10
15	3	3	6	0	6	3	4	6	3	1	3	5	7	1	20	3	6	8	0	15
16	3	7	8	3	11	3	8	9	2	6	3	9	10	1	2	3	10	10	3	20
17	3	11	11	2	16	3	13	0	1	11	3	14	1	0	7	3	15	1	3	2
18	3	16	2	1	21	3	17	3	0	16	3	18	3	3	12	3	19	4	2	7
19	4	0	5	1	4	4	1	5	3	21	4	2	6	2	17	4	3	7	1	12
20	4	4	8	0	8	4	5	8	3	3	4	6	9	1	22	4	7	10	0	17
21	4	8	10	3	13	4	9	11	2	8	4	11	0	1	4	4	12	0	3	22
22	4	13	1	2	18	4	14	2	1	13	4	15	3	0	9	4	16	3	3	4

TABLES
OF
EXCISE,
According to the
Respective Allowances
Made to
Common Brewers,
Both in
LONDON
And the
COUNTRY.

Exciſe for Common Brewers in the *Country*, at 4 *s.* 9 *d. per* Barrel, *&c.*

Bar.	*l.*	*s.*	*d.*	*f.*	Bar.	*l.*	*s.*	*d.*	*f.*
1403	296	19	10	2	2093	443	1	1	2
1426	301	17	3	0	2116	447	18	6	0
1449	306	14	7	2	2139	452	15	10	2
1472	311	12	0	0	2162	457	13	3	0
1495	316	9	4	2	2185	462	10	7	2
1518	321	6	9	0	2208	467	8	0	0
1541	326	4	1	2	2231	472	5	4	2
1564	331	1	6	0	2254	477	2	9	0
1587	335	18	10	2	2277	482	0	1	2
1610	340	16	3	0	2300	486	17	6	0
1633	345	13	7	2	2323	491	14	10	2
1656	350	11	0	0	2346	496	12	3	0
1679	355	8	4	2	2369	501	9	7	2
1702	360	5	9	0	2392	506	7	0	0
1725	365	3	1	2	2415	511	4	4	2
1748	370	0	6	0	2438	516	1	9	0
1771	374	17	10	2	2461	520	19	1	2
1794	379	15	3	0	2484	525	16	6	0
1817	384	12	7	2	2507	530	13	10	2
1840	389	10	0	0	2530	535	11	3	0
1863	394	7	4	2	5060	1071	2	6	0
1886	399	4	9	0	7590	1636	13	9	0
1909	404	2	1	2	10120	2142	5	0	0
1932	408	19	6	0	12650	2677	16	3	0
1955	413	16	10	2	15180	3213	7	6	0
1978	418	14	3	0	17710	3748	18	9	0
2001	423	11	7	2	20240	4284	10	0	0
2024	428	9	0	0	22770	4820	1	3	0
2047	433	6	4	2	25300	5355	12	6	0
2070	438	3	9	0	27830	5891	3	9	0

Excise for Common Brewers in the Country, at 1 *s.* 3 *d. per* Barrel, the Allowances being 2 ½ Barrels in every 23.

	○				¼				½				¾			
l.	*s.*	*d.*	*f.* 23	*l.*	*s.*	*d.*	*f.* 23	*l.*	*s.*	*d.*	*f.* 23	*l.*	*s.*	*d.*	*f.* 23
1	1	1	1 11	0	1	4	2 19	0	1	8	0 5	0	2	11	1 1
2	2	2	2 22	0	2	6	0 7	0	2	9	1 16	0	3	0	3 1
3	3	4	0 19	0	3	7	1 18	0	3	10	3 4	0	4	2	0 15
4	4	5	1 21	0	4	8	3 6	0	4	0	0 15	0	5	3	2 6
5	5	6	3 5	0	5	10	0 17	0	6	1	2 3	0	6	4	3 11
6	6	8	0 2	0	6	11	2 5	0	7	2	3 14	0	7	5	0 21
7	7	9	2 8	0	8	0	3 16	0	8	4	1 2	0	8	7	2 10
8	8	10	3 19	0	9	2	1 4	0	9	5	2 13	0	9	8	3 21
9	10	0	1 7	0	10	3	2 15	0	10	7	0 1	0	10	10	1 5
10	11	1	2 18	0	11	5	0 3	0	11	8	1 12	0	11	11	2 20
11	12	3	0 6	0	12	6	1 14	0	12	9	3 0	0	13	1	0 8
12	13	4	1 17	0	13	7	3 2	0	13	11	0 11	0	14	2	1 19
13	14	5	3 5	0	14	9	0 13	0	15	0	1 22	0	15	3	3 7
14	15	7	0 16	0	15	10	2 1	0	16	1	3 10	0	16	4	0 1
15	16	8	2 4	0	16	11	3 12	0	17	3	0 21	0	17	5	1 22
16	17	9	3 15	0	18	1	1 0	0	18	4	2 9	0	18	7	3 17
17	18	11	1 3	0	19	2	2 11	0	19	5	3 20	0	19	9	1 5
18	1	0	2 14	1	0	3	3 22	1	0	7	1 8	0	0	10	2 16
19	1	1	4 0	1	1	5	1 10	1	1	8	2 19	0	2	0	4 4
20	1	2	5 1	13	1	2	6	2	21	1	2	10	0 7	1	3	1	1 15
21	1	3	6 3	1	1	3	8	0	9	1	3	11	1 18	1	4	2	3 3
22	1	4	7 0	12	1	4	9	1	20	1	5	0	3 6	1	5	4	0 14

Excise for Common Brewers in the *Country*, at 1 *s.* 3 *d. per* Barrel, &c.

Bar.	*l.*	*s.*	*d.*	*f.*	Bar.	*l.*	*s.*	*d.*	*f.*
23	1	5	7	2	713	39	14	4	2
46	2	11	3	0	736	41	0	0	0
69	3	16	10	2	759	42	5	7	2
92	5	2	6	0	782	43	11	3	0
115	6	8	1	2	805	44	16	10	2
138	7	13	9	0	828	46	2	6	0
161	8	19	4	2	851	47	8	1	2
184	10	5	0	0	874	48	13	9	0
207	11	10	7	2	897	49	19	4	2
230	12	16	3	0	920	51	5	0	0
253	14	1	10	2	943	52	10	7	2
276	15	7	6	0	966	53	16	3	0
299	16	13	1	2	989	55	1	10	2
322	17	18	9	0	1012	56	7	6	0
345	19	4	4	2	1035	57	13	1	2
368	20	10	0	0	1058	58	18	9	0
391	21	15	7	2	1081	60	4	4	2
414	23	1	3	0	1104	61	10	0	0
437	24	6	10	2	1127	62	15	7	2
460	25	12	6	0	1150	64	1	3	0
483	26	18	1	2	1173	65	6	10	2
506	28	3	9	0	1196	66	12	6	0
529	29	9	4	2	1219	67	18	1	2
552	30	15	0	0	1242	69	3	9	0
575	32	0	7	2	1265	70	9	4	2
598	33	6	3	0	1288	71	15	0	0
621	34	11	10	2	1311	73	0	7	2
644	35	17	6	0	1334	74	6	3	0
667	37	3	1	2	1357	75	11	10	2
690	38	8	9	0	1380	76	17	6	0

Excise for Common Brewers in the *Country*, at 1 *s.* 3 *d. per* Barrel, &c.

Bar.	*l.*	*s.*	*d.*	*f.*	Bar.	*l.*	*s.*	*d.*	*f.*
1403	78	3	1	2	2093	116	11	10	0
1426	79	8	9	0	2116	117	17	6	0
1449	80	14	4	2	2139	119	3	1	2
1472	82	0	0	0	2162	120	8	9	0
1495	83	5	7	2	2185	121	14	4	2
1518	84	11	3	0	2208	123	0	0	0
1541	85	16	10	2	2231	124	5	7	2
1564	87	2	6	0	2254	125	11	3	0
1587	88	8	1	2	2277	126	16	10	2
1610	89	13	9	0	2300	128	2	6	0
1633	90	19	4	2	2323	129	8	1	2
1656	92	5	0	0	2346	130	13	9	0
1679	93	10	7	2	2369	131	19	4	2
1702	94	16	3	0	2392	133	5	0	0
1725	96	1	10	2	2415	134	10	7	2
1748	97	7	6	0	2438	135	16	3	0
1771	98	12	1	2	2461	137	1	10	2
1794	99	18	9	0	2484	138	7	6	0
1817	101	4	4	2	2507	139	13	1	2
1840	102	10	0	0	2530	140	18	9	0
1863	103	15	7	2	5060	281	17	6	0
1886	105	1	3	0	7590	422	16	3	0
1909	106	6	10	2	10120	563	15	0	0
1932	107	12	6	0	12650	704	13	9	0
1955	108	18	1	2	15180	845	12	6	0
1978	110	3	9	0	17710	986	11	3	0
2001	111	9	4	2	20240	1127	10	0	0
2024	112	15	0	0	22770	1268	8	9	0
2047	114	0	7	2	25300	1409	7	6	0
2070	115	6	3	0	27830	1550	6	3	0

Excise for Common Brewers in the *Country*, at 1 *s.* 3 *d. per* Barrel, &c.

Bar.	*l.*	*s.*	*d.*	*f.*	Bar.	*l.*	*s.*	*d.*	*f.*
23	1	5	7	2	713	39	14	4	2
46	2	11	3	0	736	41	0	0	0
69	3	16	10	2	759	42	5	7	2
92	5	2	6	0	782	43	11	3	0
115	6	8	1	2	805	44	16	10	2
138	7	13	9	0	828	46	2	6	0
161	8	19	4	2	851	47	8	1	2
184	10	5	0	0	874	48	13	9	0
207	11	10	7	2	897	49	19	4	2
230	12	16	3	0	920	51	5	0	0
253	14	1	10	2	943	52	10	7	2
276	15	7	6	0	966	53	16	3	0
299	16	13	1	2	989	55	1	10	2
322	17	18	9	0	1012	56	7	6	0
345	19	4	4	2	1035	57	13	1	2
368	20	10	0	0	1058	58	18	9	0
391	21	15	7	2	1081	60	4	4	2
414	23	1	3	0	1104	61	10	0	0
437	24	6	10	2	1127	62	15	7	2
460	25	12	6	0	1150	64	1	3	0
483	26	18	1	2	1173	65	6	10	2
506	28	3	9	0	1196	66	12	6	0
529	29	9	4	2	1219	67	18	1	2
552	30	15	0	0	1242	69	3	9	0
575	32	0	7	2	1265	70	9	4	2
598	33	6	3	0	1288	71	15	0	0
621	34	11	10	2	1311	73	0	7	2
644	35	17	6	0	1334	74	6	3	0
667	37	3	1	2	1357	75	11	10	2
690	38	8	9	0	1380	76	17	6	0

Excise

Excise for Common Brewers in the *Country*, at 1 *s.* 3 *d. per* Barrel, *&c.*

Bar.	*l.*	*s.*	*d.*	*f.*	Bar.	*l.*	*s.*	*d.*	*f.*
1403	78	3	1	2	2093	116	11	10	0
1426	79	8	9	0	2116	117	17	6	0
1449	80	14	4	2	2139	119	3	1	2
1472	82	0	0	0	2162	120	8	9	0
1495	83	5	7	2	2185	121	14	4	2
1518	84	11	3	0	2208	123	0	0	0
1541	85	16	10	2	2231	124	5	7	2
1564	87	2	6	0	2254	125	11	3	0
1587	88	8	1	2	2277	126	16	10	2
1610	89	13	9	0	2300	128	2	6	0
1633	90	19	4	2	2323	129	8	1	2
1656	92	5	0	0	2346	130	13	9	0
1679	93	10	7	2	2369	131	19	4	2
1702	94	16	3	0	2392	133	5	0	0
1725	96	1	10	2	2415	134	10	7	2
1748	97	7	6	0	2438	135	16	3	0
1771	98	12	1	2	2461	137	1	10	2
1794	99	18	9	0	2484	138	7	6	0
1817	101	4	4	2	2507	139	13	1	2
1840	102	10	0	0	2530	140	18	9	0
1863	103	15	7	2	5060	281	17	6	0
1886	105	1	3	0	7590	422	16	3	0
1909	106	6	10	2	10120	563	15	0	0
1932	107	12	6	0	12650	704	13	9	0
1955	108	18	1	2	15180	845	12	6	0
1978	110	3	9	0	17710	986	11	3	0
2001	111	9	4	2	20240	1127	10	0	0
2024	112	15	0	0	22770	1268	8	9	0
2047	114	0	7	2	25300	1409	7	6	0
2070	115	6	3	0	27830	1550	6	3	0

Excise for Common Brewers in *London*, at 4 *s.* 9 *d. per* Barrel, the Allowances being 3 Barrels in every 23 of Beer.

B.	0					¼					½					¾				
	l.	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23
1	0	4	1	8	6	0	5	1	3	19	0	6	2	1	9	0	7	2	2	22
2	0	8	3	0	12	0	9	3	2	2	0	10	3	3	15	0	11	4	1	5
3	0	12	4	2	18	0	13	5	0	8	0	14	5	1	21	0	15	5	3	11
4	0	16	6	1	1	0	17	6	2	14	0	18	7	0	4	0	19	8	1	17
5	1	0	7	3	7	1	1	8	0	20	1	2	8	2	10	1	3	9	0	0
6	1	4	9	1	13	1	5	9	3	3	1	6	10	0	16	1	7	10	2	6
7	1	8	10	3	19	1	9	11	1	9	1	10	11	2	22	1	12	0	0	12
8	1	13	0	2	2	1	14	0	3	15	1	15	1	1	5	1	16	1	2	18
9	1	17	2	0	8	1	18	2	1	21	1	19	2	3	11	2	0	3	1	1
10	2	1	3	2	14	2	2	4	0	4	2	3	4	1	17	2	4	4	3	7
11	2	5	5	0	20	2	6	5	2	10	2	7	6	0	2	2	8	6	1	13
12	2	9	6	3	3	2	10	7	0	16	2	11	7	2	6	2	12	7	3	19
13	2	13	8	1	9	2	14	8	2	22	2	15	9	0	12	2	16	9	2	2
14	2	17	9	3	19	2	18	10	1	5	2	19	10	2	18	3	0	11	0	8
15	3	1	11	1	21	3	2	11	3	11	3	4	0	1	13	3	5	0	2	14
16	3	6	1	0	4	3	7	1	1	17	3	8	1	3	7	3	9	2	0	20
17	3	10	2	2	10	3	11	3	0	0	3	12	3	1	13	3	13	3	3	3
18	3	14	4	0	16	3	15	4	2	6	3	16	4	3	19	3	17	5	1	9
19	3	18	5	2	22	3	19	6	0	12	4	0	6	2	2	4	1	6	3	15
20	4	2	7	1	5	4	3	7	2	18	4	4	8	0	8	4	5	8	1	21
21	4	6	8	3	11	4	7	1	1	1	4	8	9	2	14	4	9	10	0	4
22	4	10	10	1	17	4	11	10	3	7	4	12	11	0	20	4	13	11	2	10

Excise

Excise for Common Brewers in *London*, at 4 *s.* 9 *d.* *per* Barrel, *&c.* Beer.

Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*
23	4	15	713	147	5	1403	289	15	2093	432	5
46	9	10	736	152	0	1426	294	10	2116	437	0
69	14	5	759	156	15	1449	299	5	2139	441	15
92	19	0	782	161	10	1472	304	0	2162	446	10
115	23	15	805	166	5	1495	308	15	2185	451	5
138	28	10	828	171	0	1518	313	10	2208	456	0
161	33	5	851	175	15	1541	318	5	2231	460	15
184	38	0	874	180	10	1564	323	0	2254	465	10
207	42	15	897	185	5	1587	327	15	2277	470	5
230	47	10	920	190	0	1610	332	10	2300	475	0
253	52	5	943	194	15	1633	337	5	2323	479	15
276	57	0	966	199	10	1656	342	0	2346	484	10
299	61	15	989	204	5	1679	346	15	2369	489	5
322	66	10	1012	209	0	1702	351	10	2392	494	0
345	71	5	1035	213	15	1725	356	5	2415	498	15
368	76	0	1058	218	10	1748	361	0	2438	503	10
391	80	15	1081	223	5	1771	365	15	2461	508	5
414	85	10	1104	228	0	1794	370	10	2484	513	0
437	90	5	1127	232	15	1817	375	5	2507	517	15
460	95	0	1150	237	10	1840	380	0	2530	522	10
483	99	15	1173	242	5	1863	384	15	5060	1045	0
505	104	10	1196	247	0	1886	389	10	7590	1567	10
529	109	5	1219	251	15	1909	394	5	10120	2090	0
552	114	0	1242	256	10	1932	399	0	12650	2612	10
575	118	15	1265	261	5	1955	403	15	15180	3135	0
598	123	10	1288	266	0	1978	408	10	17710	3657	10
621	128	5	1311	270	15	2001	413	5	20240	4180	0
644	133	0	1334	275	10	2024	418	0	22770	4702	10
667	137	15	1357	280	5	2047	422	15	25300	5225	0
690	142	10	1380	285	0	2070	427	10	27830	5747	10

Excise for Common Brewers in *London*, at 4 *s.* 9 *d. per* Barrel, the Allowances being 2 Barrels in every 22 of *Ale*.

	0				¼				½				¾			
	l.	*s.*	*d.*	*f.* 22	*l.*	*s.*	*d.*	*f.* 22	*l.*	*s.*	*d.*	*f.* 22	*l.*	*s.*	*d.*	*f.* 22
1	0	4	3	3 6	0	5	4	3 2	0	6	5	2 20	0	7	6	2 16
2	0	8	7	2 12	0	9	8	2 8	0	10	9	2 4	0	11	10	2 0
3	0	12	11	1 18	0	14	0	1 14	0	15	1	1 10	0	16	2	1 6
4	0	17	3	1 2	0	18	4	0 20	0	19	5	0 16	1	0	6	0 12
5	1	1	7	0 8	1	2	8	0 4	1	3	9	0 0	1	4	9	3 18
6	1	5	10	3 14	1	6	11	3 10	1	8	0	3 6	1	9	1	3 2
7	1	10	2	2 20	1	11	3	2 16	1	12	4	2 12	1	13	5	2 8
8	1	14	6	2 4	1	15	7	2 0	1	16	8	1 18	1	17	9	1 14
9	1	18	10	1 10	1	19	11	1 6	2	1	0	1 2	2	2	1	0 20
10	2	3	2	0 16	2	4	3	0 12	2	5	4	0 8	2	6	5	0 4
11	2	7	6	0 0	2	8	6	3 18	2	9	7	3 14	2	10	8	3 10
12	2	11	9	3 6	2	12	10	3 2	2	13	11	2 20	2	15	0	2 16
13	2	16	1	2 12	2	17	2	2 8	2	18	3	2 4	2	19	4	2 0
14	3	0	5	1 18	3	1	6	1 14	3	2	7	1 10	3	3	8	1 6
15	3	4	9	1 2	3	5	10	0 20	3	6	11	0 16	3	8	0	0 12
16	3	9	1	0 8	3	10	2	0 4	3	11	3	0 0	3	12	3	3 18
17	3	13	4	3 14	3	14	5	3 10	3	15	6	3 6	3	16	7	3 2
18	3	17	8	2 20	3	18	9	2 16	3	19	10	2 12	4	0	11	2 8
19	4	2	0	2 4	4	3	1	2 0	4	4	2	1 18	4	5	3	1 14
20	4	6	4	1 10	4	7	5	1 6	4	8	6	1 2	4	9	7	0 20
21	4	10	8	0 16	4	11	9	0 12	4	12	10	0 8	4	13	11	0 4

Exciſe for Common Brewers in *London*, at 4 *s.* 9 *d. per* Barrel, *&c. Ale.*

Bar	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*
22	4	15	682	147	5	1342	289	15	2002	432	5
44	9	10	704	152	0	1364	294	10	2024	437	0
66	14	5	726	156	15	1386	299	5	2046	441	15
88	19	0	748	161	10	1408	304	0	2068	446	10
110	23	15	770	166	5	1430	308	15	2090	451	5
132	28	10	792	171	0	1452	313	10	2112	456	0
154	33	5	814	175	15	1474	318	5	2134	460	15
176	38	0	836	180	10	1496	323	0	2156	465	10
198	42	15	858	185	5	1518	327	15	2178	470	5
220	47	10	880	190	0	1540	332	10	2200	475	0
242	52	5	902	194	15	1562	337	5	2222	479	15
264	57	0	924	199	10	1584	342	0	2244	484	10
286	61	15	946	204	5	1606	346	15	2266	489	5
308	66	10	968	209	0	1628	351	10	2288	494	0
330	71	5	990	213	15	1650	356	5	2310	498	15
352	76	0	1012	218	10	1672	361	0	2332	503	10
374	80	15	1034	223	5	1694	365	15	2354	508	5
396	85	10	1056	228	0	1716	370	10	2376	513	0
418	90	5	1078	232	15	1738	375	5	2398	517	15
440	95	0	1100	237	10	1760	380	0	2420	522	10
462	99	15	1122	242	5	1782	384	15	4840	1045	0
484	104	10	1144	247	0	1804	389	10	7260	1567	10
506	109	5	1166	251	15	1826	394	5	9680	2090	0
528	114	0	1188	256	10	1848	399	0	12100	2612	10
550	118	15	1210	261	5	1870	403	15	14520	3135	0
572	123	10	1232	266	0	1892	408	10	16940	3657	10
594	128	5	1254	270	15	1914	413	5	19360	4180	0
616	133	0	1276	275	10	1936	418	0	21780	4702	10
638	137	15	1298	280	5	1958	422	15	24200	5225	0
660	142	10	1320	285	0	1980	427	10	26620	5747	10

Excise for Common Brewers in *London*, at 1 *s.* 3 *d. per* Barrel, the Allowances being 3 Barrels in every 23 *Small Beer.*

B.	0					¼					½					¾				
	l.	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23
1	0	1	1	0	4	0	1	4	1	5	0	1	7	2	6	0	2	0	3	7
2	0	2	2	0	8	0	2	5	1	9	0	2	8	2	10	0	2	11	3	11
3	0	3	3	0	12	0	3	6	1	13	0	3	9	2	14	0	4	0	3	15
4	0	4	4	0	16	0	4	7	1	17	0	4	10	2	18	0	5	1	3	19
5	0	5	5	0	20	0	5	8	1	21	0	5	11	2	22	0	6	3	0	0
6	0	6	6	1	1	0	6	9	2	2	0	7	0	3	3	0	7	4	0	4
7	0	7	7	1	5	0	7	10	2	6	0	8	1	3	7	0	8	5	0	8
8	0	8	8	1	9	0	8	11	2	10	0	9	2	3	11	0	9	6	0	12
9	0	9	9	1	13	0	10	0	2	14	0	10	3	3	15	0	10	7	0	16
10	0	10	10	1	17	0	11	1	2	18	0	11	4	3	19	0	11	8	0	20
11	0	11	11	1	21	0	12	2	2	22	0	12	6	0	0	0	12	9	1	1
12	0	13	0	2	2	0	13	3	3	3	0	13	7	0	4	0	13	10	1	5
13	0	14	1	2	6	0	14	4	3	7	0	14	8	0	8	0	14	11	1	9
14	0	15	2	2	10	0	15	5	3	11	0	15	9	0	12	0	16	0	1	13
15	0	16	3	2	14	0	16	6	3	15	0	16	10	0	16	0	17	1	1	17
16	0	17	4	2	18	0	17	7	3	19	0	17	11	0	20	0	18	2	1	21
17	0	18	5	2	22	0	18	9	0	0	0	19	0	1	1	0	19	3	2	2
18	0	19	6	3	3	0	19	10	0	4	1	0	1	1	5	1	0	4	2	6
19	1	0	7	3	7	1	0	11	0	8	1	1	2	1	9	1	1	5	2	10
20	1	1	8	3	11	1	2	0	0	12	1	2	3	1	13	1	2	6	2	14
21	1	2	9	3	15	1	3	1	0	16	1	3	4	1	17	1	3	7	2	18
22	1	3	10	3	19	1	4	2	0	20	1	4	5	1	21	1	4	8	2	22

Excise

Exciſe for Common Brewers in *London*, a
1 *s.* 3 *d. per* Bar. *&c. Small-Beer.*

Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*
23	1	5	713	38	15	1403	76	5	2093	113	15
46	2	10	736	40	0	1426	77	10	2116	115	0
69	3	15	759	41	5	1449	78	15	2139	116	5
92	5	0	782	42	10	1472	80	0	2162	117	10
115	6	5	805	43	15	1495	81	5	2185	118	15
138	7	10	828	45	0	1518	82	10	2208	120	0
161	8	15	851	46	5	1541	83	15	2231	121	5
184	10	0	874	47	10	1564	85	0	2254	122	10
207	11	5	897	48	15	1587	86	5	2277	123	15
230	12	10	920	50	0	1610	87	10	2300	125	0
253	13	15	943	51	5	1633	88	15	2323	126	5
276	15	0	966	52	10	1656	90	0	2346	127	10
299	16	5	989	53	15	1679	91	5	2369	128	15
322	17	10	1012	55	0	1702	92	10	2392	130	0
345	18	15	1035	56	5	1725	93	15	2415	131	5
368	20	0	1058	57	10	1748	95	0	2438	132	10
391	21	5	1081	58	15	1771	96	5	2461	133	15
414	22	10	1104	60	0	1794	97	10	2484	135	0
437	23	15	1127	61	5	1817	98	15	2507	136	5
460	25	0	1150	62	10	1840	100	0	2530	137	10
483	26	5	1173	63	15	1863	101	5	5060	275	0
506	27	10	1196	65	0	1886	102	10	7590	412	10
529	28	15	1219	66	5	1909	103	15	10120	550	0
552	30	0	1242	67	10	1932	105	0	12650	687	10
575	31	5	1265	68	15	1955	106	5	15180	825	0
598	32	10	1288	70	0	1978	107	10	17710	962	10
621	33	15	1311	71	5	2001	108	15	20240	1100	0
644	35	0	1334	72	10	2024	110	0	22770	1237	10
667	36	5	1357	73	15	2047	111	5	25300	1375	0
690	37	10	1380	75	0	2070	112	10	27830	1512	10

The foregoing Tables (though very ſhort) will ſhew the exact Duty for any Number of Barrels; and their uſe will be render'd very plain and eaſie, by the Directions and Example following.

1. If the number of Barrels of ſtrong or ſmall Beer given be leſs than 23, ſeek it in the firſt Column of the Table, and againſt it you have the exact Duty in Pounds, Shillings, Pence, Farthings, and 23th parts of a Farthing. Thus by the firſt Table the Duty of 10 Barrels and a half will be found to be 2 *l.* 4 *s.* 5 *d.* 1 *f.* and $\frac{8}{23}$ parts of a Farthing; for againſt 10 in the Column of Barrels, and under $\frac{1}{2}$ you have this Sum.

2. For any Number above 23, ſeek it in ſome of the Columns of Barrels in the Table, and if you cannot find the ſame Number, take the next leſs, and ſet down the Sum of Money againſt it, then ſubtract the Number found from the Number given, the Remainder (which will ever be leſs than 23) being found in the firſt Column of the Table, gives you a Sum of Money, which added to the former is the exact Duty required.

Example;

Example.

What Sum of Money will the Duty of 1306½ *Barrels of Strong Beer amount unto?*

See the Work by the firſt Table.

Number given ——	1306 ½	*l.*	*s.*	*d.*	*f.*
Next leſs *per* Table—	1288 =	272 :	13 :	00 :	0 : 00
Remainder ——————	18 ½ =	3 :	18 :	3 :	3 : 12
The exact Duty is————		276 :	11 :	3 :	3 : 12

That is 276 *l.* 11 *s.* 3 *d.* 3 *f.* and ½½ parts of a Farthing.

And ſo for any other Number of Barrels by this or any of the other Tables.

SECT.

SECT. XI.

Short RULES FOR *Gauging of Malt.*

THE Duty upon Malt is charged upon the Buſhel, and ſo proportionably for a greater or leſſer Quantity. The Buſhel here intended is the *Winchester* Buſhel, which (as ſettled by the firſt Act for laying a Duty upon Malt) contains 2150.42, ſolid Inches, but in Practice 2150, will be ſufficiently near the Truth.

The Area of any plain Figure may be found in Inches, by the Rules laid down in *Section* III. Now if the Area in Inches be divided by 2150, the Quotient ſhews the Buſhels and Parts which ſuch Figure or Veſſel contains at one Inch deep.

Example. There is a *Ciſtern* or *Uting Fat*, whoſe Baſe is a *Rectangular Parallelogram*, the Length is 72 Inches, and the Breadth 48 Inches, what is the Area or Content at one Inch deep? *Anſwer*, 1.607, that is, one Buſhel, and .607 thouſand Parts of a Buſhel. For 72 multiplied by 48, is 3456; and this divided by 2150, gives 1.607.

Buſ-

But the Area of any ſuch Figure may be more readily found by the Lines A and B upon the Rule. Thus,

As 2150 upon A, is to one of the ſides upon B;
So is the other ſide upon A, to the Area upon B.

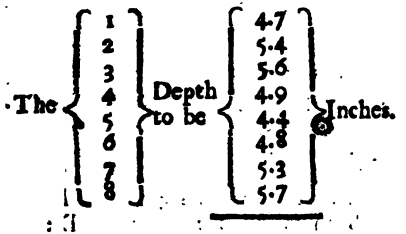
So in the Example above:

Set 2150 upon A, to 72 upon B;
Then againſt 48 upon A, is 1.607 upon B:
Or ſet 2150 upon A, to 48 upon B;
Then againſt 72 upon A, is 1.607 upon B.

The Area thus found, being multiplied by the Mean Depth in Inches, gives the Content of ſuch *Ciſtern* at that Depth; and ſo for any quantity of Malt upon the Floor. But care muſt be taken in finding the Mean Depth of any quantity of Malt, either in the Ciſtern or upon the Floor, by reaſon of the Unevenneſs of the Surface of the Malt, and of the bottom of the *Ciſtern* or Floor upon which it is laid; in order to which, take the Depth in 6, 8, or 10 Places; add all theſe Depths together, and divide the Sum by the number of Places in which the Depths were taken, the Quotient will be the Mean Depth required.

Example. Suppoſe a Quantity of Malt upon a Floor, of which you take the Depth in Eight ſeveral Places, and find,

The



The Sum of theſe is 40.8, which divided by 8, (the number of places) gives 5.1 for the Mean Depth.

Barley is ſometimes ſteep'd in Round Veſſels, and the Area of a Circle in Buſhels is thus found. Take the Diameter in Inches, and let the Square thereof be multiplied by .0003643, or divided by 2737.47, the Product or Quotient will be the Area ſought.

Example. Suppoſe the Diameter be 60 Inches, this ſquared is 3600, and this multiplied by .0003643, or divided by 2737.47, gives 1.315, the Area required.

But for any Round Veſſel, the Rule gives the Area in Buſhels by Inſpection, by help of a certain Number call'd the *Gage-Point*, which in this caſe is 52.32, (being the Diameter of a Circle, whoſe Area is 2150, the number of ſolid Inches contain'd in the Buſhel.) Thus in the Example above, the Diameter was 60 Inches; therefore, ſet the *Gage-Point*, (*viz.* 52.32) upon D, to 1 upon C, then againſt 60 upon D, is 1.315 the Area upon C. And the Rule being thus ſet, againſt any Diameter upon D, you have the Area upon C.

The

The Diameter and Depth of any Cylindrical Veſſel being given in Inches, the Content in Buſhels is readily found at once. Thus: Set the *Gage-Point* upon D, to the Depth upon C; then againſt the Diameter upon D, is the Content upon C.

In the laſt Example the Diameter was 60, ſuppoſe the Depth 40 Inches, the Content will be 52.6 Buſhels, and ſo for any other.

Having above declared what the Content of the *Winchester* Buſhel is, I ſhall here, for the ſatisfaction of the Inquiſitive Reader, give an Account how the Content of the ſaid Buſhel came to be aſcertained.

By the Statute of the 11 *Hen.* VII. *Cap.* 4. It is enacted, That Standard-Weights and Meaſures be made, and ſent to the ſeveral Cities, Burroughs, and Market-Towns therein mentioned, which was accordingly done. But the Statute of the 12 *Hen.* VII. *Cap.* 5. recites, that upon more diligent Examination, the ſaid Standard-Weights and Meaſures were found to be defective. And Enacts, That the Meaſure of a Buſhel ſhall contain 8 Gallons of Wheat, and that every Gallon contain 8 Pounds *Troy* of Wheat, and that every Pound contain 12 Ounces *Troy* Weight, and every Ounce contain 20 *Sterlings* [now 20 Penny Weight] and every *Sterling* [or Penny Weight] be of the Weight of 32 Corns of Wheat, that grew in the middle of the Ear of Wheat. And that a Standard of a Buſhel and a Gallon, after this Aſſize be made, and kept in the King's Treaſury for ever. And new Buſhels and Gallons were ordered to be made, and ſent to the ſeveral Cities, Burroughs, *&c.* and the old ones brought and broken.

Accord-

Accordingly there remains now in the Cuſtody of the Chamberlains of the *Exchequer* a Standard Braſs Buſhel, and a Standard Gallon agreeing thereto. Upon the Buſhel there is this Inſcription.

Henricus Septimus Dei Gratiâ Rex Hanglie & Francie.

In *February* 1696. when a Bill was depending in Parliament, for laying a Duty upon *Malt*, I did, with the aſſiſtance of Mr. *George Tollet*, Mr. *Phil. Shales*, and Mr. *Tho. Jett*, and in the Preſence of ſeveral Members of the Houſe of Commons, make an Experiment in order to find the true Content of the ſaid Standard Buſhel in the *Exchequer*: And the ſaid Braſs Buſhel being exactly filled with common Spring-Water, and the ſaid Water being meaſured out again by a Regular Parallelepipedon (whoſe Baſe was four Inches ſquare, and the Depth 14 Inches) was found to contain 2145.6 ſolid Inches. The ſaid Water was alſo weighed by the Standard Weights in the *Exchequer*, (and by a Beam which would turn with ſix Grains, with thirty Pounds in each Scale) and found to be 1131 Ounces and 14 Penny Weight *Troy*.

Now to ſettle the Dimenſions for a Buſhel to anſwer the ſaid Standard, theſe were agreed upon as moſt convenient, *viz.* 18.5 Inches Diameter, and 8 Inches deep; for a Cylindrical Veſſel of theſe Dimenſions will contain 2150.42 ſolid Inches, which exceeding the Content of the Standard Buſhel but 4.82 Inches, and there being no other convenient Dimenſions (without counting to the hundredth part of an Inch) that would come ſo near as theſe: It was Enacted, in the Act for laying Duty upon *Malt*,

That

That every Round Buſhel, with a plain and even Bottom, being eighteen Inches and a half Diameter throughout, and eight Inches deep, ſhall be eſteemed a legal Winchester *Buſhel, according to the Standard in His Majeſty's Exchequer.*

When this Experiment was made at the *Exchequer*, we alſo compared the Standard *Troy* Weights with the Standard *Averdupoiſe* Weights, and found that 15 Pound of *Averdupoiſe*, were equal to 18 Pound 2 Ounces, and 15 Penny-weight *Troy*; ſo that 240 Ounces *Averdupoiſe*, are equal to 218.75 Ounces *Troy*, which in the leaſt Terms in whole Numbers are,

Averd.	*Troy*	
192	175	Ounces.
144	175	Pounds.

The Buſhel, as now ſettled, contains 2150.42 ſolid Inches, as was ſhewn above, and will contain of common Spring-Water 1134.344 Ounces *Troy*. For,

As 2145.6, the Inches in the Braſs Standard, is to 1131.7 the *Troy* Ounces of Water it contained;
So is 2150.42, the Content of a Buſhel, as now ſettled, to 1134.344.

Hence the following Table ſhews the Content, in ſolid Inches, of a Buſhel, half Buſhel, Peck, and Gallon, and likewiſe the weight of Water that each will contain.

Inches

	Inches.	*Troy.* lb.	ou.	p.	w.	*Averdupoise.* lb.	ou.	dr.
Bushel.	2150.42	94.	6.	6.	88	77.	12.	8. 60
Half Bush.	1075.21	47.	3.	3.	44	38.	14.	4. 30
Peck.	537.60	23.	7.	11.	72	19.	7.	2. 15
Gallon.	268.80	11.	9.	15.	86	9.	11.	9. 07

By this Table it is eaſie to make any of the Meaſures therein mentioned, or to try ſuch as are already made, by the help of a good pair of Scales, and true weight But where theſe are not to be had, the following Table ſhews the Diameters and Depths that will anſwer to the Buſhel, half Buſhel, Peck, and Gallon, provided they be equally wide from the Top to the Bottom.

Example. To find whether a Buſhel be equal to the Standard, or not, ſuppoſe the Diameter be

Buſhel

Bushel.		Half Bu.		Peck.		Gallon.	
Diam.	Depth	Diam.	Depth	Diam.	Depth	Diam.	Depth
17.5	8.94	13.7	7.30	10.7	5.98	8.3	4.97
17.6	8.84	13.8	7.20	10.8	5.87	8.4	4.85
17.7	8.74	13.9	7.09	10.9	5.76	8.5	4.74
17.8	8.64	14.0	6.98	11.0	5.66	8.6	4.63
17.9	8.55	14.1	6.89	11.1	5.56	8.7	4.52
18.0	8.45	14.2	6.79	11.2	5.46	8.8	4.42
18.1	8.35	14.3	6.69	11.3	5.36	8.9	4.32
18.2	8.27	14.4	6.60	11.4	5.27	9.0	4.23
18.3	8.18	14.5	6.51	11.5	5.18	9.1	4.13
18.4	8 09	14.6	6.42	11 6	5.09	9.2	4.04
18.5	8.00	14.7	6.34	11.7	5.00	9.3	3.96
18.6	7.91	14.8	6.25	11.8	4.92	9.4	3.87
18.7	7.83	14.9	6.17	11.9	4.83	9.5	3.79
18.8	7.75	15.0	6.08	12.0	4.75	9.6	3.71
18.9	7.67	15.1	6.00	12.1	4.68	9.7	3.64
19.0	7.58	15.2	5.93	12.2	4.60	9.8	3.56
19.1	7.50	15.3	5.85	12.3	4.52	9.9	3.49
19.2	7.42	15.4	5.77	12.4	4.45	10.0	3.42
19.3	7.35	15.5	5.70	12.5	4.38	10.1	3.36
19.4	7.27	15.6	5.63	12.6	4.31	10.2	3.29
19.5	7.20	15.7	5.56	12.7	4.24	10.3	3.22

19 Inches, and one tenth part of an Inch, find this in the firſt Column, and againſt it you have 7.5, that is, ſeven Inches and a half; now if the Buſhel be exactly ſo deep, it is a legal Buſhel, if the Depth be more, it may be cut to this Depth. If the Buſhel be not exactly round, and the Difference of the Diameters be not above an Inch,

add the longeſt and ſhorteſt Diameters together, and take half the Sum for the Mean Diameter, and proceed as above.

Suppoſe the Diameter of a Buſhel be 19.5 Inches, and the Depth but 7 Inches, you will find by the Table the Depth ſhould be 7.2 Inches, from whence you may conclude this Buſhel is leſs than the Standard; now to find how much it wants, ſay,

As 7.2 Inches, the Depth in the Table, is to 64, the Pints in a Buſhel,

So is 7 Inches, the Depth of the Buſhel given, to 62.22.

That is 62 Pints, and almoſt a quarter of a Pint, which ſubtracted from 64, the Remainder 1.77, that is, one Pint and three Quarters, and ſomewhat better, and ſo much this Buſhel is too little.

Or thus, ſubtract 7, the depth of the Buſhel given, from 7.2, the Depth found in the Table, the Difference is .2; then as 7.2 : 64 : : 2 : 1.777 as above.

The Uſe of the Tables for Half Buſhels, Pecks, and Gallons, are the ſame with reſpect to thoſe Meaſures; and therefore Examples of theſe are needleſs.

SECT.

SECT. XII.

The USE of the RULE In Queſtions of Simple Intereſt.

Problem. I.

To find the Intereſt of any Sum of Money for any time propoſed (in Years, Months, or Days) at any Rate of Intereſt.

1. *For Years.*

Exam. WHat is the Intereſt of 350 *l.* for five Years, at 6 *per Cent. per Ann.*?

1. Set 1 upon A, to .06 *the Rate* upon B, then againſt 5, *the Time* upon A, is .3, *the Intereſt* of 1 *l. in five Years* upon B.

2. Set 1 upon A, to 3 upon B, then againſt 350, *the Sum given*, upon A, is 105 upon B, that is 105 *l.* the Intereſt of 350 *l.* in five Years, which was required.

2. *For Months.*

Exam. WHat is the Intereſt of 350 *l.* for four Months, at 6 *per Cent. per Ann.*?

1. Set 12 upon A, to .06 *the Rate upon* B, then againſt 4, *the Months given* upon A, is .02, *the Intereſt of* 1 l. *in four Months* upon B.

2. Set 1 upon A, to .02 upon B, then againſt 350 *the Sum given* upon A, is 7 upon B; that is, 7 *l.* the Intereſt ſought.

3. *For Days.*

Exam. WHat is the Intereſt of 350 *l.* for ſeventy three days, at 6 *per Cent per Ann.*?

1. Set 365 upon A, to .06 *the Rate* upon B, then againſt 73 *the days given* upon A, is .012 *the Intereſt of* 1 *l. in* 73 *days* upon B.

2. Set 1 upon A, to .012 upon B, then againſt 350 *the Sum* given upon A, is 4.2 upon B; that is, 4 *l.* 4 *s.* the Intereſt ſought.

There are ſeveral other uſeful Queſtions relating to *Simple Intereſt*, ſome of which ſhall be reſolved in the following Problems. In order to which, put

u = A *Principal* or Sum forborn.
t = The *Time* of Forbearance in Years, or parts of a Year.
g = The Gain or *Rate of Interest.*
a = The *Amount* of the said Principal, in the said Time, and at the said Rate of Interest.

Then if any three of these Particulars be given, the fourth may be found.

Problem II.

Given u, g *and* t: *To find* a.

$$1 : g\,t + 1 :: u : a.$$

Exam. WHat will 250 *l.* amount unto in 6 Years, at 6 *per Cent. per Annum*?

u = 250, g = .06. t = 6, and
g t + 1 = 1.36, *Therefore,*

Set 1 upon A, to 1.36 upon B, then against 250 upon A, is 340 upon B, the *Amount* sought.

Problem III.

Given a, g *and* t: *To find* u.

$$g\,t + 1 :: 1 : a : u.$$

Exam WHat is 340 *l.* due 6 Years hence *worth* in ready Money, at 6 *per Cent. per Ann.*

$a = 350$, $g = .06$, $t = 6$
and $g\,t + 1 = 1.36$ *Therefore*,

Set 136 upon A, to 1 upon B, then againſt 340 upon A, is 250 upon B, the *Preſent Worth* ſought.

Problem IV.

Given a, u *and* g: *To find* t.

$$u\,g : 1 :: a - u : t.$$

Examp. IN how many Years will 250 *l.* amount unto 340 *l.* at 6 *per Cent. per Ann.*?

$a = 340$, $u = 250$, $g = .06$
$u\,g = 15$, *and* $a - u = 90$.

Therefore, Set 15 upon A, to 1 upon B, then againſt 90 upon A, is 6 upon B, that is, 6 Years the *Time* ſought.

Pro-

Problem V.

Given a, u *and* t. *To find* g.

u t : 1 : : a — u : g.

Exam. AT what *Rate of Intereſt* will 250 *l.* Amount unto 340 *l.* in 6 Years?

u = 340, a = 250, t = 6
u t = 1500 *and* a — u = 90

Therefore, Set 1500 upon A, to 1 upon B, then againſt 90 upon A, is .06 upon B, which is the *Rate* of *Intereſt* ſought.

The *Intereſt* of Money for *Days*, requiring more Exactneſs than can be expected from any Inſtrumental Operations, here is added a ſhort Table, which readily ſhews the *Intereſt* of any *Sum* for any *Time*, and at any *Rate of Intereſt*, to the hundredth part of a Farthing, and is alſo very uſeful for other purpoſes.

A Table of *Simple Intereſt* for any Sum, Time, and Rate.

	l.	s.	d	f.		s.	d.	f.
1000000	2739	14	6	0.99	90	4	11	0.71
900000	2465	15	0	3.29	80	4	4	2.41
800000	2191	15	7	1.59	70	3	10	0.11
700000	1917	16	1	3.89	60	3	3	1.81
600000	1643	16	8	2.19	50	2	8	3.51
500000	1369	17	3	0.49	40	2	2	1.21
400000	1095	17	9	2.79	30	1	7	2.90
300000	821	18	4	1.09	20	1	1	0.60
200000	547	18	10	3.40	10	0	6	2.30
100000	273	19	5	1.70				
90000	246	11	6	0.33	9	0	5	3.67
80000	219	3	6	0.96	8	0	5	1.04
70000	191	15	7	1.59	7	0	4	2.41
60000	164	7	8	0.22	6	0	3	3.78
50000	136	19	8	2.85	5	0	3	1.15
40000	109	11	9	1.48	4	0	2	2.52
30000	82	3	10	0.11	3	0	1	3.89
20000	54	15	10	2.74	2	0	1	1.26
10000	27	7	11	1.37	1	0	0	2.63
9000	24	13	1	3.23	.90	0	0	2.37
8000	21	18	4	1.10	.80	0	0	2.10
7000	19	3	6	2.96	.70	0	0	1.48
6000	16	8	9	0.82	.60	0	0	1.58
5000	13	13	11	2.68	.50	0	0	1.32
4000	10	19	2	0.55	.40	0	0	1.05
3000	8	4	4	2.41	.30	0	0	0.79
2000	5	9	7	0.27	.20	0	0	0.53
1000	2	14	9	2.14	.10	0	0	0.26
900	2	9	3	3.12	.09	0	0	0.24
800	2	3	10	0.11	.08	0	0	0.21
700	1	18	4	1.10	.07	0	0	0.18
600	1	12	10	2.08	.06	0	0	0.16
500	1	7	4	3.07	.05	0	0	0.13
400	1	1	11	0.05	.04	0	0	0.11
300	0	16	5	1.04	.03	0	0	0.08
200	0	10	11	2.03	.02	0	0	0.05
100	0	5	5	3.01	.01	0	0	0.02

The uſe of the Table.

I. To find the *Intereſt* of any *Sum*, for any number of *Days*, at any *Rate* of *Intereſt*.

Multiply the *Sum* given by the *Rate*, and that Product by the number of *Days*, then ſet down the Figures of the laſt Product, with Cyphers towards the right Hand, as in the Example following, the Sums anſwering to theſe Numbers taken out of the Table, and added together give the *Intereſt* ſought.

1. *Example.* What is the *Intereſt* of 6525 *l.* for 146 *Days*, at 8 *per Cent. per Ann.*?

Principal ——— 6525
Rate ——— .08

1. Product ——— 522.00
Number of Days — 146

3132
2088
522

2. Product ——— 76212

	l.	*s.*	*d.*	*f.*
70000 ———	191.	15.	7.	1. 59
6000 ———	16.	8.	9.	0. 82
200 ———	0.	10.	11.	2. 03
10 ———	0.	0.	6.	2. 30
2 ———	0.	0.	1.	1. 26
Anſwer ———	208.	16.	0.	0. 00

2. *Example.* What is the *Intereſt* of 825 *l.* 10 *s.* in 146 *Days*, at 8 *per Cent. per Ann.*

Principal	825. 5
Rate	.08
1. Product	66 04
Num. of Days	146
	39624
	26416
	6604
2. Product	9641.84

	l	*s.*	*d.*	*f.*
9000	24.	13.	1.	3. 23
600	1.	12.	10.	2. 08
40	0.	2.	2.	1. 21
1	0.	0.	0.	2. 63
.80	0.	0.	0.	2. 10
.04	0.	0.	0.	0. 11
Anſwer	26.	8.	3.	3. 36

3. *Example.* What is the *Intereſt* of 825.5 *l.* at the ſame *Rate*, for the Remainder of the Year, *viz.* 219 *Days?*

1. Product	66.04
Number of Days	219
	59436
	6604
	13208
2. Product	14462.76

10000

	l.	*s.*	*d.*	*f.*
10000	27.	7.	11.	1. 37
4000	10.	19.	2.	0. 55
400	1.	1.	11.	0. 05
60	0.	3.	3.	1. 81
2	0.	0.	1.	1. 26
.70	0.	0.	0.	1. 84
.06	0.	0.	0.	0. 16

The *Intereſt* of 825.5 *l.* for 219 *Days*, at 8 *per Cent.* is — 39. 12. 5. 3. 04

The *Intereſt* of 825.5 *l.* for 146 *Days*, at the ſame Rate, was found to be — 26. 8. 3. 3. 36

The Sum is the exact *Intereſt* of 825.5 *l.* for 365 *Days*, or one *Year* at 8 *per Cent. per Ann.* viz. — 66. 0. 9. 2. 40

II. *Any Annual Salary, or Penſion, being given, to find what it amounts to by the Day.*

Againſt any Annual Salary in the firſt or third Col. of the Table, you have the Amount for one Day, in the ſecond or fourth Col. reſpectively. Thus,

		l.	*s.*	*d.*	*f.*	
5000	Pounds	13.	13.	11.	2. 68	
600	*per An.*	1.	12.	10.	2. 08	*per* Day.
70	is	0.	3.	10.	0. 11	

And

And ſo for any other Sum, found in the firſt or third Column of the Table.

And ſo for any number of Days; multiply the Annual Salary by the number of Days given, and place the Figures of the Product, with Cyphers towards the right Hand, as in the Example following, the Sums anſwering to theſe Numbers taken out of the *Table*, and added together, give the Anſwer.

Example. At a Salary of 525 *l. per Ann.* what is due for 73 Days?

Salary ——— 525
Days ——— 73

1575
3675

38325

30000	———	82.	3.	10.	0.	11
8000	———	21.	18.	4.	1.	10
300	———	0.	16.	5.	1.	04
50	———	0.	0.	3.	1.	15
20	———	0.	1.	1.	0.	60
Anſwer		105.	0.	0.	0.	00

Of

Of Annuities at Simple Intereſt.

Problem VI.

To find the Amount of any Annuity for any Time given, and at any Rate of Simple Intereſt.

A General Rule.

1. *Find the Intereſt of the Annuity for half the number of Years, half Years or Quarters, that the Annuity is to continue.*

2. *Multiply the Intereſt thus found by the number of Years, half Years, or Quarters, leſs by* 1, *and to this Product add the Sum of the Annual, half yearly, or quarterly Payments, the Sum will be the true Amount of the Annuity.*

Example. *What is the Amount of an Annuity of* 100 l. *in* 8 *Years, at* 6 *per Cent.?*

The Intereſt of 100 *l.* for half the time, *viz.* 4 Years, is 24 *l.* this multiplied by the number of Years leſs by 1, (*viz.* 7.) the Product will be 168, to which add the Sum of the Annual Payments, *viz.* 800, the Sum will be 968 *l.* the Amount ſought.

Pro-

Problem VII.

To find the preſent worth of any Annuity for any given Time, at any Rate, accompting Simple Intereſt.

The Rule, or Proportion is,

As the Amount of 1 l. *in any Time is to* 1 l. *So is the Amount of any Annuity, to the preſent worth.*

Therefore, Divide the Amount of the Annuity by the Amount of one Pound in the Time given, the Quotient is the preſent Worth.

Example. *What is the preſent Worth of an Annuity of* 100 l. *to continue* 8 *Years, at* 6 *per Cent.?*

The Amount of 100 *l.* Annuity for 8 Years, by the laſt *Problem*, was found to be 968 *l.* and the Amount to one Pound in 8 Years, will be found (by the II *Problem* before going) to be 1.48.

Now if 968 (the Amount of the Annuity) be divided by 148, the Quotient is 654.054, and ſo much preſent Money is equal in Value to an Annuity of 100 *l.* for 8 years; for if this 654.054 be put out at Intereſt, it will in 8 Years amount to 968 *l.* at 6 *per Cent. per Ann.* Several Writers have treated of the Calculation of Annuities at *Simple Intereſt*; but in as much as all *preſent Worths* either of *Principal Sums*, or *Annuities* due at any Time to come, ought to be conſider'd in a *Geometrical Proportion*, according

according to the ſeveral *Rates* of *Compound Intereſt*, all Computations of *Annuities* at *Simple Intereſt*, are to be laid aſide as uſeleſs.

Of Compound Intereſt.

IN Compound Intereſt, the reſpective Amounts for each reſpective Year are ſo many *Geometrical proportional* Numbers. Thus, if 100 *l.* be put out to Intereſt at 6 *per Cent. per Annum*, there will be due at the end of the firſt Year 106 *l.* which 106 is now made a Principal, and at the end of the ſecond Year will amount to 112.36, and this being made a Principal, will at the end of the third Year be increaſed to be 119.1016; now theſe four Numbers, *viz.* 100, 106, 112.36, and 119.1016, are four *Geometrical proportional* Numbers: *For*,

As 100 to 106, (for the firſt Year);
So is 106 to 112.36 (for the ſecond Year.)

Again, *As* 106 to 112.36;
So is 112.36 to 119.1016, for the third Year, *&c.*

Pro-

Problem VIII.

Any Sum of Money being propounded, to find the Amount thereof in any time, at any Rate of Compound Intereſt.

Example: *What is the Amount of* 100 l. *being forborn* 5 *Years, at* 6 per Cent. per Ann. *Compound Intereſt?*

1. SET 100 upon B, to 106 (the firſt Year's Amount) upon A; then againſt 106 upon B, you have 112.36 upon A, the Amount in two years. *In like manner,*

2. Againſt 112.36 upon B, is 119.1016 upon A, the Amount in three Years. *Again,*

3. Againſt 119.1016 upon B, is 126.247 upon A, the Amount in four Years.

Laſtly, Againſt 126.247 upon B, you have 133.822 upon A, which is the Amount in 5 Years, and the Anſwer to the Queſtion.

Problem IX.

A Sum of Money being due at any time to come, to find what it is worth in ready Money, at any Rate of Intereſt.

Example: *What is the preſent worth of* 133.822 *l. due at the end of* 5 *Years to come, at* 6 per Cent. per Annum *Compound Intereſt?*

THIS is but the Inverſe of the laſt Problem: Therefore if you ſet the Rule as is there directed, you may read the Proportion back again: For the reſpective Amounts found upon A, in that Example, will point you to the reſpective preſent Worth at the end of each reſpective Year.

Thus, Set 106 upon A, to 100 upon B; then againſt 133.822 upon A, is 126.247 upon B: The preſent Worth of 100 *l.* at the end of the firſt Year.

Alſo, Againſt 126.247 upon A, you have 119.1016 upon B, the preſent Worth at the end of the ſecond Year.

Again, Seek 119.1016 upon A, againſt it you have 112.36 upon B, the preſent Worth at the end of the third Year.

In like manner, Againſt 112.36 upon A, is 106 upon B, the preſent Worth at the end of the fourth Year.

Laſtly, Againſt 106 upon A, you have 100 upon B, which is the preſent Worth of 133.822 *l.* at the end of five Years, which was required.

Pro-

Problem X.

A yearly Rent or Annuity being forborn a certain number of Years, to find what the Arrearages thereof will amount unto at any Rate propounded.

1. FInd a Sum (or Principal) whoſe Intereſt is equal to the Annuity or yearly Rent.
2. Find (by *Probl.* VIII.) the Amount of that Sum in the time given: This done, ſubtract the Principal firſt found from the Amount, the Remainder will be the Anſwer.

Example: *A Rent or Annuity of* 9 l. per Annum *being forborn* 22 *Years, what will the Arrearages thereof amount unto, accompting Intereſt upon Intereſt, at* 6 per Cent.?

To find a Principal whoſe Intereſt is 9 *l. ſay,*

As 6 to 100;
So is 9 to 150, the Principal ſought.

The Amount of 150 *l.* in 12 Years, will be found by Problem VIII. to be 301.828, from which ſubtract (the Principal firſt found, *viz.*) 150, the Remainder will be 151.828, the Sum of the Arrearages ſought.

Pro-

Problem XI.

A yearly Rent or Annuity being propounded, to find what it is worth in ready Money?

FIrſt, find (by the laſt *Problem*) what the Arrearages thereof will amount unto at the end of the Term propounded: Then (by *Probl.* IX.) find the preſent Worth of thoſe Arrearages: The preſent Worth thus found is the value of the Annuity or Yearly Rent ſought.

Example: *Suppoſe a Rent of* 9 l. per Ann. *to continue* 12 *Years, What Sum of Money may be given for this Rent, allowing the Buyer* 6 per Cent. *Compound Intereſt?*

By the laſt Problem the Arrearages of a Rent of 9 *l. per Ann.* being forborn 12 Years, were found to be 151.828 *l.*

And the preſent Worth of 151.828, due at the end of 12 Years to come, will be found (by *Prob.* IX.) to be 75.443, and ſo much may be given for a Rent of 9 *l. per Ann.* to continue 12 Years.

But ſuppoſe the Term of the Rent or Annuity begins not till after the Expiration of certain Years to come, in ſuch caſe, find what the Arrearages forborn for all that time are worth in ready Money, for that is the Anſwer.

For Inſtance.

Suppoſe in the laſt Example, the Annuity of 9 *l.* for 12 Years, were not to begin till after the Expiration of 6 Years: I here ſeek what the Arrearages, (*viz.* 151.828) being forborn 18 Years are worth in ready Money, and by *Problem* IX. I find it will be 53.185 *l.* which is the Anſwer: So a Yearly Rent or Annuity of 9 *l. per Annum*, to begin 6 Years hence, and continue 12 Years, is worth but 53.185, in ready Money.

Problem XII.

A Sum of Money being propounded, to find what Annuity (to continue any Number of Years, and at any Rate propoſed) that Sum will buy.

TAke any Annuity at pleaſure, and find the Value thereof in ready Money: This done, the Proportion is,

As *the preſent Worth found, is to the Annuity taken*; So *is the Sum given, to the Annuity ſought.*

Example: *What Annuity to continue* 12 *Years will* 300 l. *Purchaſe, allowing the Buyer* 6 per Cent. *Compound Intereſt?*

By Problem XI. I find that 75.443 will purchaſe 9 *l. per Annum*, to continue 12 Years.

Therefore, Set 75.443 upon B, to 9 upon A; then againſt 300 upon B, is 35.776 upon A, that is 35 *l.* 15 *s.* 6 *d*; and ſo much Annuity to continue 12 Years is worth 300 *l.*

Pro-

Problem XIII.

There is a Freehold Eſtate of the yearly Value of 78 l. *What Sum of Money is this Eſtate worth, allowing the Buyer* 6 per Cent. per Ann. *Compound Intereſt for his Money?*

The Rule or Proportion is,

As *the Annual, half Yearly or Quarterly Intereſt of* 1 l. *is to* 1 l.;
So *is the Annual, half Yearly, or Quarterly Rent to the Sum ſought.*

The Intereſt of 1 *l.* at 6 *per Cent.* Compound Intereſt,

$$\text{For } 1 \left\{\begin{matrix}\text{Year}\\ \text{Half Year}\\ \text{Quarter}\end{matrix}\right\} \text{ is } \left\{\begin{matrix}.06\\ .02956\\ .01467\end{matrix}\right\}$$

Now to Anſwer the Queſtion.

Set .06 upon B, to 1 upon A; then againſt 78 upon B, is 1300 upon A, which is the Value of the Eſtate propounded; and for the ſame reaſon I find without moving the Rule, that a Freehold Eſtate

$$\text{of } \left\{\begin{matrix}90\\ 120\\ 150\end{matrix}\right\} \text{ Pounds } \textit{per Ann.} \text{ is worth } \left\{\begin{matrix}1500\\ 2000\\ 2500\end{matrix}\right\} \text{ Pounds.}$$

But

But ſuppoſe the Rent of the former Eſtate, (*viz.* 78 *l.*) were payable *Quarterly*, (*viz.* 19.5 *l. per* Quarter:) *What it is then worth*, allowing the Purchaſer 6 *per Cent.* as before?

Set .01467 (the Intereſt of 1 *l.* for a Quarter of a Year) upon A, to 1 upon B; then againſt 19.5 (the Quarter's Rent) upon A, you have 1329.244 upon B, that is 1329 *l.* 4 *s.* 10 $\frac{1}{2}$ *d.* the Anſwer, which is 29 *l.* 4 *s.* 10 $\frac{1}{2}$ *d.* more than the former; that being accounted by Yearly, this by Quarterly Payments.

Problem XIV.

A Sum of Money propounded, to find the Annual Rent of a Free-hold Eſtate that ſuch a Sum will purchaſe, at a given Rate of Compound Intereſt.

THis is the Inverſe of the laſt; and therefore the Proportion is:

As 1 *to the Annual Intereſt*;
So *is the Sum propounded to the Annual Rent.*

Example: *Suppoſe I would lay out* 3650 l. *upon a Free-hold Eſtate, ſo as to have* 6 per Cent. *Comp. Intereſt for my Money, What muſt the Annual Rent be?*

Set 1 upon A, to .06 upon B; then againſt 3650 upon A, you have 219 upon B: The Anſwer is 219 *l. per Annum.*

Pro-

Problem XV.

The Annual Rent of a Freehold Eſtate being known, and alſo the Sum of Money which was paid for the ſame, to find what Rate of Compound Intereſt the Purchaſer ſhall have for his Money.

The Proportion is,

As *the Sum given is to* 1 ;
So *is the Annual Rent to the Rate of Intereſt.*

Example: *There is* 1300 l. *given for a Freehold Eſtate, whoſe Annual Rent is* 78 l. *What Rate of Intereſt has the Purchaſer for his Money?*

Set 1300 upon A, to 1 upon B; then againſt 78 upon A, is .06 upon B: So the Purchaſer ſhall have *6 per Cent.* for his Money.

If it be demanded, *How many Years Purchaſe a Freehold Eſtate is worth at any Rate of Compound Intereſt?*

The Proportion is,

As *the Rate of Intereſt is to* 1 ;
So *is* 1 *to the Anſwer.*

Example, at 5 per Cent.

Set .05 upon B, to 1 upon A; then againſt 1 upon B, is 20 upon A; ſo at 5 *per Cent.* it is worth 20 Years Purchaſe.

At

At 6 *per Cent.* it is worth but 16.66 Years Purchaſe. *For,*

As .06 *upon* B, *to* 1 *upon* A;
So *is* 1 *upon* B, *to* 16.66 *upon* A; *that is,* 16 *Years* 8 *Months, and ſomewhat more.*

The Converſe of this is, *to find by the Number of Years Purchaſe, what Rate of Intereſt the Buyer hath for his Money.*

Say, As *the Number of Years Purchaſe is to* 1;
So *is* 1 *to the Rate of Intereſt.*

Example: *A Freehold Eſtate is bought at* 20 *Years Purchaſe; What Rate of Intereſt hath the Purchaſer for his Money?*

Set 20 upon A, to 1 upon B, then againſt 1 upon A, is .05 upon B; that is, 5 *per Cent. per Ann.*

SECT.

SECT. XIII.

The Rule of Fellowſhip.

Problem I.

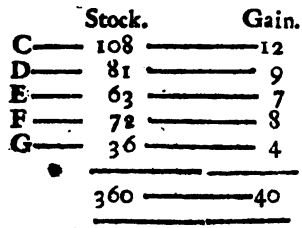
Five Perſons C D E F *and* G, *make a Bank, or Stock of Money, in all* 360 l. *with which they Trade and gain* 40 l. *what part of this Gain muſt each Perſon receive to balance his Money put in?*

THE Inſtrument will reſolve this and ſuch like Queſtions by Inſpection.

The Proportion is,

As the whole Stock,
is to the whole Gain;
So *is each Man's particular Stock,*
to his particular Gain.

Therefore, Set 360 the whole Stock upon A, to 40 the whole Gain upon B, then againſt each Perſon's Stock upon A, you have his Gain upon B. Thus againſt 108 upon A, you have 12 upon B. So C who put 108 *l.* ſhall have 12 *l.* of the Gain, and ſo of the reſt.



By which it appears, that this Queſtion (which by the Pen would require five Operations) is reſolved by the Rule at one.

A Queſtion of this Rule with Time.

Problem II.

Two Merchants join together, H *put in* 60 l. *for* 6 *Months,* F *put in* 80 l. *for* 8 *Months, and they gained* 50 l; *How much muſt each Merchant have of the Profit?*

The Proportion will be,

(1) As 1 *is to* 6 *(Months)*;
So *is* 60 l. *to* 360.

(2) As 1 *is to* 8 *(Months)*;
So *is* 80 l. *to* 640.

Set 1 upon A, to 6 upon B; then againſt 60 upon A, is 360 upon B.

Again,

Again, Set 1 upon A, to 8 upon B; then againſt 80 upon A, is 640 upon B, the Sum of theſe is 1000.

Now for each Merchant's Share.

1. Set 1000 upon A, to 50 upon B; then againſt 360 upon A, is 18 upon B, that is 18 *l.* and ſo much is H's Share.

2. Set 1000 upon A, to 50 upon B; then againſt 640 upon A, is 32 upon B, which is the Share belonging to F.

So that $\left\{\begin{matrix} H \\ F \end{matrix}\right\}$ muſt have $\left\{\begin{matrix} 18\ l. \\ 32\ l. \end{matrix}\right.$

The whole Gain 50 *l.*

SECT. XIV.

The Uſe of the RULE IN *Military* AFFAIRS.

Problem I.

Any number of Men being propoſed, to place them in Battalia, ſo that there may be as many in Rank as in File, and ſuppoſing them to be imbattelled at Order (which is 3 Foot diſtance in Rank and File) to find how many ſquare Foot of Ground they will take up?

Example. LET the Number of Men propoſed be 3600. Set the Lines C and D even at the end. Now foraſmuch as the Number given doth conſiſt of an even Number of Figures, ſeek the Number, *viz.* 3600 in the ſecond Radius on the Line C, and againſt it on the Line D, you will

will find 60, and ſo many Men muſt be in Rank, and as many in File:

Now to find how many ſquare Foot of Ground they will occupy:

Set 1 upon A to 3 upon B, then againſt 60 upon A is 180 upon B.

Then ſet the Lines C and D even, and find this Number, *viz.* 180 on the firſt Radius on the Line D, and againſt it you have 32400 upon C, and ſo many ſquare Foot of Ground will theſe Soldiers occupy, being at their Order of three Foot.

Problem II.

Any Number of Men being propoſed, to place them in Battalia, in ſuch order that there may be twice as many in Rank as in File, and that they may ſtand in cloſe Order, which is 1 ½ Foot.

Example. LET the Number given be 1800, ſet the Lines D and C even, and count half the Number given, *viz.* 900 upon the Line C, and againſt it you have 30 upon D, which is the Depth of the File, and then there muſt be 60 in Rank, which is twice 30.

Now for the Ground theſe will occupy being at cloſe Order.

Set 1 upon A to 1.5 upon B, then againſt 30 upon A is 45 the ſide of the Ground, ——— and without moving the Lines, ſeek 60 upon the Line

A, and againſt it you have 90 upon B, which is the Length of the Ground.

Now for the Area.

Set 1 upon A, to 45 upon B; then againſt 90 upon A, is 4050, and ſo many ſquare Foot of Ground will theſe 1800 Men occupy at Cloſe Order.

Problem III.

Any Number of Men being propoſed to be put in Battalia, and a certain Number named to be either in Rank or File, to find the other Number.

Example. LET it be required to place 1200 Men in Battalia, ſo that there ſhall be 15 in File, how many muſt there be in Rank; or how many Files muſt there be?

This is reſolv'd by the Lines A and B. And the Proportion is,

As 15 *the Depth in File, is to* 1;

So *is* 1200, *the Number propoſed, to* 80 *the Number of Men in Rank.*

Pro-

Problem IV.

Any Number of Soldiers being given, together with their diſtance in Rank and File, to order them into a Square Battalia of Ground.

Example. LET the Number of Soldiers given be 5400, their Diſtance in File 6 Foot, and in Rank 4 Foot.

Set 6 upon the Line A, to 4 upon the Line B, and then againſt 5400 upon A, is 3600 upon B.

Then ſet the Lines D and C even, and ſeek 3600 in the ſecond Radius upon C, againſt it you have 60 upon D, the number of Men to be placed in File.

Now for the number of Men in Rank, ſay,

As 60 *is to* 1;
So is 5400 *to* 90.

So there ſhall be 60 in File, and 90 in Rank.

Now if you multiply 60, the Number of Men in File, by 6, their Diſtance, the Product will be 360, the ſide of your Ground.

In like manner, if you multiply 90, the Number of Men in Rank, by 4, their Diſtance, the Product will be 360, as above; which ſhews, your Ground is exactly ſquare, and is therefore a Proof of the Work.

Problem V.

If 1000 *Soldiers may be Lodged or Quartered in a Square Piece of Ground, whoſe Side is* 400 *Foot; how many Foot long muſt the Side of a Square be, that the Ground included may lodge* 4000?

The Proportion will be

(1) As 1 is to 400 (the Side of the Square which will Lodge 1000 Soldiers)
So is 4000, to 160000.

(2) As 1000 is to 40000;
So is 160000 to 640000.

Now (ſetting the Lines D and C even) find this 640000 in the ſecond Radius on the Line C, and againſt it you have 800 upon D, and ſo much muſt the Side of a Square be, to Lodge 4000 Soldiers with the ſame convenience that a 1000 were Lodged in a Square whoſe Side was 400 Foot.

SECT.

SECT. XV.

Of Triplicate Proportion.

Triplicate Proportion, *is ſuch a Proportion as is between* Lines *and* Solids, *or between* Solids *and* Lines, *of which I ſhall give ſome Examples in the following* Problems.

Problem I.

If a Bullet whoſe Diameter is 4 Inches, weighs 9 Pounds, what ſhall another Bullet of the ſame Metal weigh, whoſe Diameter is 8 Inches?

SET 4 (the Diameter of the Bullet propoſed) upon the ſingle Line, to 9 the weight on the Line E, then againſt 8 upon the ſingle Line is 72, which is the weight of the Bullet whoſe Diameter is 8 Inches.

And as the Lines now ſtand, you may find the weight of all Bullets of the ſame Metal; for againſt the Diameter upon the ſingle Line you have the weight upon the Line E.

Thus, if the Diameter of a Bullet

be	3 5 6 7	Inches, the weight be	will	4.79 17.57 30.37 48.23	Pounds.

Problem II.

Having the Weight of two Bullets of the ſame Metal, and the Diameter of one of them, to find the Diameter of the other.

THis is but the Converſe of the laſt, and therefore without moving the Lines, find the Weight on the Line E, and againſt it you have the Diameter on the ſingle Line.

Thus, if a Bullet

weighs	20 30 40	Pounds, the Diameter will be		5.23 5.97 6.67	Inches.

Problem

Problem III.

If a Braſs Gun, whoſe Diameter is 8 *Inches, weighs* 640 *Pounds, what ſhall another Gun weigh (of the ſame Shape and Metal) whoſe Diameter is* 6 *Inches?*

SET 8 upon the ſingle Line, to 640 on the Line E, then againſt 6 upon the ſingle Line, is 270 on the Line E, which is the Weight of the Piece whoſe Diameter is 6 Inches.

Problem IV.

There is a Conical Tun which contains 620 *Gallons, the Diameter at the Bottom is* 80 *Inches, the Diameter at the Top* 71.2 *Inches, and the Depth* 30 *Inches; it is deſired to make another Tun in the ſame Shape with the former, that may contain* 800 *Gallons. To find the Diameter of this Tun at the Bottom and Top, and alſo the Depth.*

1. *For the Diameter at the Bottom.*

SET 620 (the Content of the firſt Tun) upon the Line E, to 80 (the Bottom-Diameter) on the ſingle Line: Then againſt 800 (the Content of the ſecond Tun) upon E, is 87, which is the Diameter ſought.

2. *For*

2. For the Diameter at the Top.

Set 620 upon E to 72.2 upon the ſingle Line, then againſt 800 upon E is 77.4 the Diameter ſought.

3. For the Depth.

Set 620 upon E to 30. upon the ſingle Line, then againſt 800 upon E is 32.63 the Depth required.

So the Diameters and Depth of this Tun are as followeth,

Bottom } Diameter { 87 }
Top — } { 77.4 } Inches.
Depth —— 32.63 }

But the firſt of theſe being found as above directed, the other may be diſcover'd at one Operation by the Lines A and B. Thus:

Set 80 upon A to 87 upon B, then againſt 72.2 upon A is 77.4 upon B, and againſt 30 upon A is 32.63 upon B, which alſo ſhews, that the Diameters and Depth of this Tun, are in proportion to the Diameters and Depth of the Tun firſt propoſed. And the like may be done for any Caſks; of which I need not give any Examples.

Problem

Problem V.

There is a Ship of 60 Tun Burthen, which measures 44 Foot by the Keel, and it is required to make another Ship of 360 Tun Burthen, of the same Shape with the first, How many Foot long must the Keel of the second Ship be?

SET 60 upon E, to 44 upon the single Line, then against 360 upon E, is 80 on the single Line, which is the Length of the Keel required.

Now suppose in the first Ship the middle Beam be 17.6 Foot, and the Depth of the Hold 7.76 Foot.

To find these Dimensions of the second Ship,

Set 44 upon A, to 80 upon B, then against 17.6 upon A, is 32 upon B, which is the Length of the Beam. And without moving the Lines, seek 7.76 upon A, and against it you have 14.1 upon B, which is the Depth of the Hold. So the Dimensions and Burthen of the two Ships are as followeth,

		1 Ship	2 Ship	
Length of the	Keel	44	80	Foot.
	Beam	17.6	32	
Depth of the Hold		7.76	14.1	
Burthen		60	360	Tun.

To

To find the Burthen of any Ship, whoſe Length, Breadth, and Depth is given in Feet, the Common Rule is,

Multiply the Length of the Keel by the Length of the Midſhip Beam, then multiply that Product by the Depth of the Hold; then divide this laſt Product by 100, *the Quotient gives you the Tuns. But when there is no Allowance for Guns, divide the laſt Product by* 95.

Thus, in the ſecond Ship abovementioned, multiply 80, the Length, by 32 the Breadth, the Product is 2560; this multiplied by 14.1, the Depth, gives 36096; and this divided by 100, is 360.96, that is, 360 Tons, and 96 parts of a Ton, which is the Burthen of this Ship, allowing for Guns. But if there be no Allowance made for Guns, divide the laſt Product, *viz.* 360.96, by 95, in this caſe her Burthen will be found to be 379.95, that is, 380 Ton very near.

SECT.

SECT. XVI.

PROPORTIONS IN A CIRCLE.

IT was ſhewn in Page 42, that if the *Diameter* of a Circle be 1, the *Circumference* will be 3.141592, and the *Area* .785398, the Square-Root of .785398 is .886227, and this is the Side of a Square, equal to the Area of a Circle, whoſe Diameter is 1. And the Square-Root of .5, *viz.* .707107 is the Side of the greateſt Square that can be inſcribed in a Circle, whoſe Diameter is 1: So that if the Diameter of a Circle be 1,

the	Circumference	is	3.141593
	Square equal		.886227
	Squ. inſcribed		.707107
	Area ———		.785398

[*Note,*

Note further, that if the Circumference of a Circle be 1,

The	Diameter	will be	.318310
	Sq. equal		.282095
	Sq. Inſcribed		.225079
	Area		.079577

If the Area of a Circle be 1.

The	Diameter	is	1.128379,
	Circumference		3.54491,

If the Circumference of a Circle be 4,

The	Diameter	is	1.27324,
	Area		

If the Diameter of a Circle be 4,

The	Circumference	is	12.56637.
	Area		

In all theſe Caſes the *Area* is Square Inches, Foot, Yards, &c. the other Terms being Lines, do refer to Lineal, Inches, Foot, Yards, &c. reſpectively. Theſe Things premiſed, I ſhall here add ſome uſeful Queſtions, which may be reſolved either by the Pen or Inſtrument.

1. *The*

1. *The Diameter of a Circle given, to find the Circumference.*

As 1 is to 3.141593, Or
As .31731 is to 1;
So is the Diameter to the *Circumference.*

2. *The Circumference given, to find the Diameter.*

As 1 is to .31831, Or
As 3.141593 is to 1;
So is the Circumference to the *Diameter.*

3. *The Diameter given, to find the Side of a Square, equal to the Circle.*

As 1 is to .886227, Or
As 1.128379 is to 1;
So is the Diameter to the Side of a *Square equal.*

4. *The Side of a Square given, to find the Diameter of a Circle whose Area is equal to the Square.*

As 1 is to 1.128379, Or
As .886227 is to 1;
So is the side of the Square to the *Diameter* sought.

5. *The Circumference given, to find the side of a Square equal to the Circle.*

As 1 is to .282095, Or
As 3.54491 is to 1;
So is 1 the Circumference to the side of the *Square* required.

6. *The*

6. *The Side of a Square given, to find the Circumference of a Circle equal to the Square.*

As 1 is to 3.54491, Or,
As .282095, is to 1 ;
So is the ſide of the Square to the *Circumference* ſought.

7. *The Diameter of a Circle given, to find the ſide of the greateſt Square inſcribed.*

As 1 is to .707107, Or
As 1.41421 is to 1 ;
So is the Diameter to the *Side* required.

8. *The ſide of a Square given, to find the Diameter of a Circle that ſhall circumſcribe the Square.*

As 1 is to 1.41421, Or.
As .707107 is to 1 ;
So is the Side of the Square to the *Diameter* ſought.

9. *The Circumference of a Circle given, to find the Side of the Square inſcribed.*

As 1 is to .225079, Or
As 4.4429 is to 1 ;
So is the Circumference to the *Side* required.

10. *The*

10. *The Side of a Square given, to find the Circumference of a Circle that ſhall circumſcribe the Square.*

As 1 is to 4.4429, Or
As .225079 *is* to 1;
So is the Side of the Square to the *Circumference* ſought.

11. *The Diameter of a Circle given to find the Area.*

As 1 is to .785398, Or
As 1.27324 is to 1;
So is the Square of the Diameter to the *Area* required.

12. *The Area of a Circle given, to find the Diameter.*

As 1 is to 1.27324, Or
As .785398 is to 1;
So is the Area to the Square of the Diameter, whoſe Square-Root is the *Diameter* ſought.

13. *The Circumference of a Circle given, to find the Area.*

As 1 is to .079577, Or
As 12.56636 is to 1;
So is the Square of the *Circumference* to the Area ſought.

14. *The*

14. *The Area of a Circle given, to find the Circumference.*

As 1 is to 12.56637, Or
As .079577 is to 1;
So is the Area to the Square of the Circumference, whose Square-Root is the *Circumference* required.

Such of these Proportions as relate to the finding the Area of a Circle by the Diameter or Circumference, being of most general use, and requiring, in some cases, more exactness than can be attained to by the Numbers here given. I have here added two Tables for that purpose, which were Calculated to fixteen Places, and are here Printed exactly true to ten Places in the Fractions, only the last Figure is made one more, where the eleventh Figure was above five.

A Table shewing the Area and Circumference of any Circle in Inches, Foot, Yards, &c. to any Diameter in Inches, Foot, Yards, &c. from 1 to 12, or from 10 to 120.

Dia.	Area.	Circumference.
1.0	.7853981634	3.1415926536
1.1	.9503317777	3.4557519189
1.2	1.1309733553	3.7699111843
1.3	1.3273228961	4.0840704497
1.4	1.5393804003	4.3982297150
1.5	1.7671458676	4.7123889804
1.6	2.0106192983	5.0265482457
1.7	2.2698009922	5.3407075111
1.8	2.5446900494	5.6548667765
1.9	2.8352873699	5.9690260418
2.0	3.1415926536	6.2831853072
2.1	3.4636059006	6.5973445725
2.2	3.8013271108	6.9115038379
2.3	4.1547562844	7.2256631033
2.4	4.5238934212	7.5398223686
2.5	4.9087385212	7.8539816340
2.6	5.3092915846	8.1681408993
2.7	5.7255526112	8.4823001647
2.8	6.1575216010	8.7964594301
2.9	6.6051985542	9.1106186954
3.0	7.0685834706	9.4247779608

Dia.	Area.	Circumference.
3.1	7.5476763502	9.7389372261
3.2	8.0424771932	10.0530964915
3.3	8.5529859994	10.3672557568
3.4	9.0792027689	10.6814150222
3.5	9.6211275016	10.9955742876
3.6	10.1787601976	11.3097335529
3.7	10.7521008569	11.6238928183
3.8	11.3411494795	11.9380520836
3.9	11.9459060653	12.2522013490
4.0	11.5663706144	12.5663706144
4.1	13.2025431267	12.8805298797
4.2	13.8544236023	13.1946891451
4.3	14.5220120412	13.5088484104
4.4	15.2053084423	13.8230076758
4.5	15.9043128088	14.1371669412
4.6	16.6190251375	14.4513262065
4.7	17.3494454294	14.7654854719
4.8	18.0955736847	15.0796447372
4.9	18.8574099032	15.3938040026
5.0	19.6349540849	15.7079632679
5.1	20.4282062300	16.0221225333
5.2	21.2371663383	16.3362817987
5.3	22.0618344098	16.6504410640
5.4	22.9022104447	16.9646003294
5.5	23.7582944428	17.2787595947
5.6	24.6300864041	17.5929188601
5.7	25.5175863288	17.9070781255
5.8	26.4207942167	18.2212373908
5.9	27.3397100679	18.5353966562
6.0	28.2743338823	18.8495559215

Dia.	Area.	Circumference.
6.1	29.2246656600	19.1637151869
6.2	30.1907054010	19.4778744523
6.3	31.1724531052	19.7920337176
6.4	32.1699087728	20.1061929830
6.5	33.1830724035	20.4203522483
6.6	34.2119439976	20.7345115137
6.7	35.2565235549	21.0486707791
6.8	36.3168110755	21.3628300444
6.9	37.3928065594	21.6769893098
7.0	38.4845100065	21.9911485751
7.1	39.5919214169	22.3053078405
7.2	40.7150407905	22.6194671058
7.3	41.8538681274	22.9336263712
7.4	43.0084034276	23.2477856366
7.5	44.1786466911	23.5619449019
7.6	45.3645979178	23.8761041673
7.7	46.5662571078	24.1902634326
7.8	47.7836242611	24.5044226980
7.9	49.0166993776	24.8185819634
8.0	50.2654824574	25.1327412287
8.1	51.5299735005	25.4469004941
8.2	52.8101725068	25.7610597594
8.3	54.1060794765	26.0752190248
8.4	55.4176944093	26.3893782902
8.5	56.7450173055	26.7035375555
8.6	58.0880481649	27.0176968209
8.7	59 4467869876	27.3318560862
8.8	60.8212337735	27.6460153516
8.9	62.2113885227	27.9601746169
9.0	63.6172512352	28.2743338823

Dia.	Area.	Circumference.
9.1	65.038821 9109	28.5884931477
9.2	66.476100 5500	28.9026524130
9.3	67.929087 1522	29.2168116784
9.4	69.397781 7178	29.5309709437
9.5	70.882184 2466	29.8451302091
9.6	72.382294 7387	30.1592894745
9.7	73.898113 1941	30.4734487398
9.8	75.429639 6127	30.7876080052
9.9	76.976873 9946	31.1017672705
10.0	78.539816 3397	31.4159265359
10.1	80.118466 6482	31.7300858013
10.2	81.712824 9199	32.0442450666
10.3	83.322891 1548	32.3584043320
10.4	84.948665 3531	32.6725635973
10.5	86.590147 5146	32.9867228627
10.6	88.247337 6398	33.3008821281
10.7	89.920235 7274	33.6150413934
10.8	91.608841 7781	33.9292006588
10.9	93.313155 7933	34.2433599241
10.0	95.033177 7711	34.5575191895
11.1	96.768907 7122	34.8716784548
11.2	98.520345 6166	35.1858377202
11.3	100.287491 4842	35.4999969856
11.4	102.070345 3151	35.8141562509
11.5	103.868907 1093	36.1283155163
11.6	105.683176 8668	36.4424747816
11.7	107.513154 5875	36.7566340470
11.8	109.358840 2715	37.0707933124
11.9	111.220233 9187	37.3849525777
12.0	113.097335 5292	37.6991118431

The Uſe of the foregoing Table *is obvious, for*

1. IF the Diameter of a Circle be *Inches, Foot, Yards, &c.* not exceeding 12, you have it in the firſt Column to every tenth part of an *Inch, Foot, Yard, &c.* And againſt any Diameter in the firſt Column, you have the Area in the ſecond Column, and the *Circumferenee* in the third: And the *Area* is always Square *Inches, Foot, Yards, &c.* according as the Diameter or Circumference is given in *Inches, Foot, Yards, &c.*

2. If the Diameter given be entire *Inches, Feet, Yards, &c.* not exceeding 120, you have it in the firſt Column, by taking away the Point, for by this means the Tenths are made Units; but in this caſe you muſt remove the Point two places towards the right Hand in the Area, and one place in the Circumference.

Thus if the Diameter

be 11.6 the { Area is 105.6831768668 / Circumfer. is 36.4424747816

But if the Diameter

be 116, the { Area is 10568.31768668 / Circumfer. is 364.424747816

And for the ſame Reaſon if the Diameter

be 1160, the { Area is 1056831.768668 / Circumfer. is 3644.24747816

3. If any Area in the Table be multiplied by 4, the Product will be the Area of a Circle of twice the Diameter of the Circle given, and therefore the Area of a Circle whoſe Diameter is 22.6 (though not found in the Table) may be readily obtained, for the half of 22.6 is 11.3, which found in the firſt Column, the Area againſt it is 100.287491842, this multiplied by 4, is 401.1499659368, the Area of a Circle whoſe Diameter is 22.6.

4. If the Diameter and Altitude of a *Cylinder* or *Cone* be given, multiply the Area againſt the Diameter by the Altitude, the Product will be the Content of the *Cylinder*; or by one third part of the Altitude, the Product will be the Content of the *Cone*.

5. If a *Cylinder*, *Sphere* and *Cone*, have the ſame Diameter and Altitude, their Contents are as 3, 2, 1. Suppoſe the Diameter and Altitude 12, the Area againſt 12 is 113.0973355292, this multiplied by

12	gives	1357.16802263504	Cylinder.
8		904.7786842336	Sphere.
4		452.3893421168	Cone.

So that any Area multiplied by two thirds of its Diameter, gives the Solidity of a Sphere of that Diameter.

6. If the Diameter of a Sphere be doubled, the Area againſt the Sum is the Superficial Content of the Sphere. As let the Diameter be 6, the double is 12, the Area againſt 12, is 113.0973355292, the ſuperficial Content of a Sphere whoſe Diameter is 6.

A TABLE ſhewing the *Area* and *Diameter* of any Circle, in *Inches*, *Feet*, *Yards*, &c. to any *Circumference* in *Inches*, *Feet*, *Yards*, &c. from 1 to 12 or from 10 to 120.

Circ.	Area.	Diameter.
1.0	.079577471 5	.3183098862
1.1	.0962887406	.3501408748
1.2	.1145915590	.3819718634
1.3	.1344859269	.4138028520
1.4	.1559718442	.4456338407
1.5	.1790493110	.4774648293
1.6	.2037183272	.5092958179
1.7	.2299788928	.5411268065
1.8	.2578310078	.5729577951
1.9	.2872746723	.6047887837
2.0	.3183098862	.6366197724
2.1	.3509366495	.6684507610
2.2	.3851549623	.7002817496
2.3	.4209648245	.7321127382
2.4	.4583662361	.7639437268
2.5	.4973591972	.7957747155
2.6	.5379437077	.8276057041
2.7	.5801197676	.8594366927
2.8	.6238873769	.8912676818
2.9	.6692465357	.9230986699
3.0	.7161972439	.9549296586

Circ.	Area.	Diameter.
3.1	.7647395016	.9867606472
3.2	.8148733086	1.0185916358
3.3	.8665986651	1.0504226244
3.4	.9199155711	1.0822536130
3.5	.9748240264	1.1140846016
3.6	1.0313240312	1.1459155903
3.7	1.0894155855	1.1777465789
3.8	1.1490986891	1.2095775675
3.9	1.2103733422	1.2414085561
4.0	1.2732395447	1.2732395447
4.1	1.3376972967	1.3050705333
4.2	1.4037465981	1.3369015220
4.3	1.4713874489	1.3687325106
4.4	1.5406198491	1.4005634992
4.5	1.6114437988	1.4323944878
4.6	1.6838592979	1.4642254764
4.7	1.7578663464	1.4960564651
4.8	1.8334649444	1.5278874537
4.9	1.9106550918	1.5597184423
5.0	1.9894367886	1.5915494309
5.1	2.0698100349	1.6233801495
5.2	2.1517748306	1.6552114082
5.3	2.2353311757	1.6870423968
5.4	2.3204790703	1.7188733854
5.5	2.4072185143	1.7507043740
5.6	2.4955495077	1.7825353626
5.7	2.5854720505	1.8143663512
5.8	2.6769861428	1.8461973399
5.9	2.7700917845	1.8780283285
6.0	2.8647889757	1.9098593171

Circ.	Area.	Diameter.
6.1	2.9610777162	1.9416903057
6.2	3.0589580062	1.9735212943
6.3	3.1584298457	2.0053522830
6.4	3.2594932345	2.0371832716
6.5	3.3621481728	2.0690142602
6.6	3.4663946605	2.1008452488
6.7	3.5722326977	2.1326762374
6.8	3.6796622843	2.1645072260
6.9	3.7886834203	2.1963382147
7.0	3.8992961058	2.2281692033
7.1	4.0115003406	2.2600001919
7.2	4.1252961249	2.2918311805
7.3	4.2406834587	2.3236621691
7.4	4.3576623419	2.3554931578
7.5	4.4762327745	2.3873241464
7.6	4.5963947565	2.4191551350
7.7	4.7181482880	2.4509861236
7.8	4.8414933689	2.4828171122
7.9	4.9664299992	2.5146481009
8.0	4.0929581789	2.5464790895
8.1	5.2210779081	2.5783100781
8.2	5.3507891867	2.6101410667
8.3	5.4820920148	2.6419720553
8.4	5.6149863923	2.6738030439
8.5	5.7494723192	2.7056340326
8.6	5.8855497955	2.7374650212
8.7	6.0232188213	2.7692960093
8.8	6.1624793965	2.8011269984
8.9	6.3033315212	2.8329579870
9.0	6.4457751952	2.8647889757

Circ.	Area.	Diameter.
9.1	6.5898104187	2.8966199643
9.2	6.7354371916	2.9284509529
9.3	6.8826555140	2.9602819415
9.4	7.0314653858	2.9921129301
9.5	7.1818668070	3.0239439187
9.6	7.3338597777	3.0557749074
9.7	7.4874442978	3.0876058960
9.8	7.6426203673	3.1194368846
9.9	7.7993879862	3.1512678732
10.0	7.9577471546	3.1830988618
10.1	8.1176978724	3.2149298505
10.2	8.2792401396	3.2467608391
10.3	8.4423739563	3.2785918277
10.4	8.6070993224	3.3104228163
10.5	8.7734162379	3.3422538049
10.6	8.9413247029	3.3740847935
10.7	9.1108247153	3.4059157822
10.8	9.2819162811	3.4377467708
10.9	9.4545993944	3.4695777594
11.0	9.6288740571	3.5014087480
11.1	9.8047402692	3.5332397366
11.2	9.9821980307	3.5650707253
11.3	10.1612473417	3.5969017139
11.4	10.3418882021	3.6287327025
11.5	10.5241206120	3.6605636911
11.6	10.7079445712	3.6923946797
11.7	10.8933600799	3.7242256684
11.8	11.0803671381	3.7560566570
11.9	11.2689657456	3.7878876456
12.0	11.4591559026	3.8197186342

The Uses of the Table are very plain, for

1. IF the *Circumference* of a Circle be given in *Inches*, *Foot*, *Yards*, &c. not exceeding 12, you have it in the first Column to every tenth part of an *Inch*, *Foot*, *Yard*, &c. from 1, to 12, and from 10, to 120, in intire *Inches*, *Foot*, *Yards*, &c. and in either case you have the Area in the second Column, and the Diameter in the third Column.

Thus if the Circumference of a Circle

be 10.5, the { Area is 8.7734162379 / Diam. is 3.3422538049

But if the Circumference

be 105, the { Area is 877.34162379 / Diam. is 33.422538049

2. If any Area in this Table be multiplied by 4, you have the Area of a Circle to twice the Circumference of the Circle given. Let 8.9413247029 (the Area against 10.6 Circumference) be multiplied by 4, the Product will be 35.7652988116 which is the Area of a Circle whose Circumference is 21.2.

3. If the Circumference of a *Cylinder* be 10.6 Foot, and the Altitude 16 Foot, the Area against 10.6, multiplied by 16, gives the solid Content of the *Cylinder* in Feet, and so of any other.

4. If the Circumference of a *Globe* or *Sphere* be given, the Area againſt it multiplied by 4, gives the ſuperficial Content of ſuch *Sphere*. Thus if the Circumference of a *Sphere* be 3 Foot, the Area againſt 3 is .7161972439 ; this multiplied by 4, gives 2.8647889756 the ſuperficial Content of a *Sphere*, whoſe Circumference is 3 Foot. Or thus, double the Circumference given, and find the Sum in the firſt Column, the Area againſt it is the ſuperficial Content ſought; thus twice 3 is 6, which found in the firſt Column, the Area againſt it is 2.8647889756, as above. And indeed, any Area in this Table is equal to the ſuperficial Content of a Sphere, whoſe Circumference is half the Circumference againſt ſuch Area reſpectively.

5. If the Circumference of a *Sphere* be found in the firſt Column, you have the Diameter in the third Column. Now if the Area between them be multiplied by two thirds of the Diameter, you have the Solidity of ſuch *Sphere*.

SECT.

SECT. XVII.

The Uſe of the Inſtrument in Meaſuring of Timber and Stone.

Of Round Timber.

Problem I.

The Circumference and Length of any Round Tree, being given (in Foot-meaſure) to find the Content.

RULE.

AS 3.545 upon D, is to the Length upon C; *So* is the Circumference upon D, to the Content upon C.

Example. Suppoſe the Length of a Tree be 20 Foot, and the Circumference 4 Foot, What is the Content?

Set 3.545, upon D, to 23 (the Length) upon C, then againſt 4 the Circumference upon D, you have 25.46 upon C, that is 25 Feet and almoſt an half, and ſo much is the Content.

Note, 3.545, is the Circumference of a Circle whoſe Area is 1.

Problem II.

The Circumference of a Round Tree being given in Inches, and the Length in Feet, to find the Content in Feet.

THE Rule given in the laſt Problem will ſerve here, if inſtead of 3.545, you make uſe of 42.54. (which is found by multiplying 3.545 by 12.)

Example. The Circumference is 36 Inches, and the Length 10 Feet, What is the Content?

Set 42.54, upon D, to 10 upon C, and againſt 36 upon D, is 7.16 upon C, which is the Contents ſought.

Problem III.

The Circumference of any Round Tree being given, to find how much in Length it will take to make a Solid Foot.

FIrſt in Foot-Meaſure, the Proportion is,

As the Circumference upon D, is to 1 upon C;
So is 3.545 upon D, to the Length ſought upon C.

Example. If the Circumference of a Tree be 3 Feet, How many Feet in Length will it take to make a Solid Foot?

Set

Set 3 upon D, to 1 upon C, and againſt 3.545 upon D, is 1.396 upon C, that is, 1 Foot and .396 Parts. Now, ſo many times as this is contain'd in the Length of the Tree, ſo many Feet does the Tree contain.

Secondly, In Inch-meaſure, if the Circumference be given in Inches, *Say*,

As the Circumference upon D, *is to* 12 *upon* C;
So is 42.54 *upon* D, *to the Length of Inches (upon* C) *to make a Foot.*

Example. The Circumference of a Tree is 48 Inches, How many Inches in Length will make a Foot?

Set 48 upon D, to 12 upon C, and againſt 42.54 upon D, is 9.42 upon C, that is 9 Inches and .42 Parts, the Length required.

Now ſuppoſing this Tree were 20 Foot long, the Content will be found to be 25.47, that is, 25 Foot, and almoſt a half, for 20 Foot is 240 Inches, and 9.42 Inches are contain'd 25.47, times in 240.

Note, The common way uſed for the Meaſuring of Round Timber is to girt the Tree about in the middle with a Line, and to take a fourth part of the Circumference for the Side of a Square equal to the Circles; but this is falſe, though moſt Men uſe it, for according to this Suppoſition it will take 12 Inches in Length of the Tree above-mention'd to make a Foot, and ſo the whole Tree would contain but 20 Foot, whereas the true Content is 25.47 Foot, as was ſhewn above.

Note, In all that hath been ſaid of Round Timber, we ſuppoſe the Tree to be equally thick from end to end. But ſeeing moſt Trees are leſs towards

wards the Top than at the Root, the Circumference must be taken in the middle, and if the Ends differ much, it will be the more exact, if you divide the Length into two equal parts, and take the Circumference in the middle of each, and so find the Content as if they were two Trees. These Contents added together give the Content of the whole.

For Instance, There is a Tree of 12 Foot long, the Circumference at one end is 6 Foot, and at the other end 3 Foot, the Sum of these is 9, the half is 4.5, the Circumference in the middle, by which (and the Length) the Content will be found (by Problem I.) to be 19.34 Foot.

But if this Tree be divided into two parts, as each may be 6 Foot long, the Circumference in the middle of the greater will be 5.25, by which the Content will be found to be 13.16 Foot.

And the Circumference in the middle of the lesser piece will be 3.75, and the Content of this will be 6.72, which added to the Content of the other, *viz.* 13.16, gives 19.88 Foot, the Content of the whole Tree: Whereas the Content found by one Circumference taken in the middle of the whole Tree, is but 19.34.

But in most Cases it will be sufficient to take the Circumference in the middle of the Tree, and so find the Content as in *Problem* I.

Observe also, in Taper-Trees, when it is said so much in length will make a Foot, it must be understood in that place only where the Circumference was taken, for it will not take so much in Length of the greater end of a Tree to make a Foot, as it will of the lesser.

Yet,

Yet, if by the Circumference taken in the middle you find the Length of a Foot, that Length ſo found will ſhew the whole Content of the Tree, as in Problem III.

Problem IV.

The Circumference of any Round Tree being given, either in Feet or in Inches, to find how many Solid Feet are contained in one Foot of the Length.

IF the Circumference be given in Feet, ſet 3.545 upon D, to 1 upon C; then againſt the Circumference upon D, is the Content upon C.

Example. Let the Circumference of a Round Tree be 4 Foot, How many Solid Feet are contained in one Foot of the Length of this Tree? Anſwer, 1.273:

For, As 3.545 upon D, is to 1 upon C;
So is 4 upon D, to 1.273 upon C.

If the Circumference be given in Inches, make uſe of 42.54, inſtead of 3.545, and then the Proportion will hold as above.

Of

Of Square Timber.

Problem V.

The Length and Side of any Piece of Square Timber, being given in Foot-Measure, to find the Content.

The Proportion is,

AS 1 *upon* D, *is to the Length upon* C;
So is the Side upon D, *to the Content upon* C.

Example. There is a Piece of Timber exactly Square, each ſide at either end being 1.5 Foot, and the Length 20 Foot, How many ſolid Feet doth this Piece contain?

Set 1 upon D, to 20 upon C, and againſt 1.5 upon D, is 45 upon C, which is the Content ſought.

Problem VI.

The Side of the Square ſolid being given in Inches, and the Length in Feet, to find the Content in Feet.

THE Proportion in the laſt Problem will hold here, if inſtead of 1 you make uſe of 12 for the firſt Term.

Example.

Example. Let the Side be 18 Inches, and the Length 20 Foot, What is the Content in Feet? Anſwer 45 : For,

As 12 upon D, is to 20 the Length upon C;
So is 18 the Side upon D, to 45 the Content upon C.

When the Ends are not exactly ſquare (which doth often happen) take the Breadth and Thickneſs at the end in Feet and Parts, then multiply the Breadth by the Thickneſs, and this Product by the Length, the laſt Product will be the Content.

If the Breadth and thickneſs be given in Inches, and the Length in Feet, then by the Lines A and B on the Rule,

The Proportion is,

(1.) *As* 1 *upon* B, *to the Breadth upon* A;
So is the Thickneſs upon B, *to a fourth Number, viz.* (the Area of the end in ſquare Inches.)

(2) *As* 144 *upon* B, *to the fourth Number upon* A;
So is the Length upon B, *to the Content ſought.*

But if any ſuch Solid be greater at one end than at the other, the exact way to find the Content, is to find the Area of each Baſe (in Foot-meaſure) then find a Geometrical Mean betwixt the ſaid Area's, then if the Sum of theſe three be multiplied by one third part of the Length, the Product is the Content.

SECT.

Or having the Length and Height in Feet, you may find the Content more readily on the Rule by this Proportion.

As 272.25 *upon* B, *is to the Height upon* A;
So is the Length upon B *to a fourth Number upon*
which is the true Content, if the Wall be one Brick and a half thick: If the thickneſs be more or leſs, *ſay,*

As 3 *upon* B, *is to the half Bricks thick upon* A;
So is the fourth Number above found upon B, *to the Content required.*

Example. There is a Wall 16 Foot high, and 97 Foot long: How many ſquare Rod doth this Wall contain at one Brick and a half thick? Anſwer 5 Rod and 7 tenth parts: For,

As 272.25 *upon* B, *is to* 16 *the Height upon* A;
So is 97 *the Length upon* B, *to* 5.7 *upon* A.

Now if the Wall above-mention'd were two Bricks and a half thick, it would contain 9 Rod and an half: For,

As 3 *upon* B, *is to* 5 *(the half Bricks in two and a half) upon* A;
So is 5.7 *the Content at one Brick and a half thick upon* B, *to* 9.5 *the Content ſought upon* A.

But if a Wall of 16 Foot high, and 97 Foot long be but 1 Brick thick, the Content would be but 3 Rod and 8 tenth parts: For,

As

SECT. XVIII.

The Uſe of the Rule in the Menſuration of Brick-work.

BRick-work is commonly meaſured by the Rod-Square, which is 16.5 Feet in Length, and as much in Breadth, and by conſequence each ſquare Rod doth contain 272.25 ſquare Foot, for 16.5 multiplied by 16.5 is 272.25.

Note alſo, all Brick-work muſt be reduced to the Standard-meaſure of one Brick and a half thick.

Problem I.

The Length and Height of any Brick-Wall being given in Feet, to find the Content in ſquare Rods at any thickneſs.

The RULE.

MUltiply the Height by the Length, and divide the Product by 272.25, the Quotient will be the true Content, if the Wall be juſt one Brick and a half thick:

If the thickneſs be more or leſs: *Then ſay,*

As 3, *is to the Number of half Bricks in the Thickneſs of the Wall;*

So is the Quotient above found, to the true Content ſought.

Or

Or having the Length and Height in Feet, you may find the Content more readily on the Rule by this Proportion.

As 272.25 *upon* B, *is to the Height upon* A;
So is the Length upon B *to a fourth Number upon* A, which is the true Content, if the Wall be one Brick and a half thick: If the thickneſs be more or leſs, *ſay,*

As 3 *upon* B, *is to the half Bricks thick upon* A;
So is the fourth Number above found upon B, *to the Content required.*

Example. There is a Wall 16 Foot high, and 97 Foot long: How many ſquare Rod doth this Wall contain at one Brick and a half thick? Anſwer 5 Rod and 7 tenth parts: For,

As 272.25 *upon* B, *is to* 16 *the Height upon* A;
So is 97 *the Length upon* B, *to* 5.7 *upon* A.

Now if the Wall above-mention'd were two Bricks and a half thick, it would contain 9 Rod and an half: For,

As 3 *upon* B, *is to* 5 *(the half Bricks in two and a half) upon* A;
So is 5.7 *the Content at one Brick and a half thick upon* B, *to* 9.5 *the Content ſought upon* A.

But if a Wall of 16 Foot high, and 97 Foot long were but 1 Brick thick, the Content would be but 3 Rod and 8 tenth parts: For,

As

As 3 *upon* B, *is to* 2 (*the half Bricks in one*) *upon* A;

So is 5.7 (*the Content at one and a half thick*) *upon* B. *to* 3.8 *the Content upon* A.

Now, that (in all these Cases) you may find the Content at one Operation: I shall here lay down certain fix'd Numbers for any Thickness hereafter express'd,

viz. at	1	Bricks thick	408.37
	1 ½		272.25
	2		204.19
	2 ½		163.35
	3		136.12
	3 ½		116.68
	4		102.10
	4 ½		90.75
	5		81.75

By these Numbers the Content of any Wall, at any Thickness here mentioned may be found either by the Pen or Rule, and the Proportion is,

As *the Number proper to the Thickness, is to the Height*;

So *is the Length to the Content.*

Example. There is a Wall four Bricks and a half thick, 7 Foot high, and 85 Foot long: How many square Rod doth this Wall contain?

The proper Number for four Bricks and a half thick, is 90.75.

Therefore,

Therefore, Set 90.75 upon A, to 7 the Height upon B; then againſt 84 the Length upon A, is 6.555 upon B; that is, ſix Rod and a half, and ſomething more.

Note, The Decimal parts of a Rod may be reduced into Feet by this Proportion.

As 1.00 *upon* A. *is to* 272.25 *upon* B;
So *is any Decimal upon* A, *to the Number of Feet upon* B.

And thus you may find the Decimal in the laſt Example, *viz.* .555 is equal to 151 Foot.

Alſo, $\left\{\begin{matrix}\frac{1}{4}\text{ or }.25\\ \frac{1}{2}\text{ or }.50\\ \frac{3}{4}\text{ or }.75\end{matrix}\right\}$ of a Rod is $\left\{\begin{matrix}66\\ 136\\ 204\end{matrix}\right\}$ Feet.

Problem II.

The Height of any Wall being given in Feet, to find how many Feet in Length it will take to make a Square Rod at any Thickneſs above-mention'd.

RULE.

SET the Height upon A, to the Number proper for the Thickneſs upon B, then againſt 1 upon A, is the Length ſought upon B.

Example

Example. There is a Wall one Brick and half thick, and 8 Foot high: How many Feet in Length will it take to make a ſquare Rod? Anſwer, 34. For,

The proper Number for one and a half thick is 272.25. *Therefore,*

As 8 *(the Height) upon* A, *is to* 272.25 *upon* B;
So *is* 1 *upon* A, *to* 34 *upon* B;

SECT. XIX.

The Uſe of the Rule in the Menſuration of Superficies, as Board, Paving, Cieling, Wainſcotting, Flooring, Tyling, &c.

Note, In all Square or Oblong Superficies the Area or Content may be found by this Proportion:

As 1 *upon* A, *to the Breadth upon* B;
So *is the Length upon* A, *to the Content upon* B.

This Proportion will hold in Triangular Superficies alſo, if you take half the Perpendicular for the Breadth, and the Side upon which it falls for the Length: And in all Caſes, if the Sides be given

in { Inches / Feet / Yards } the Area will be Square { Inches, / Feet, or / Yards. }

But

But in many Caſes the Sides are given in one kind of Meaſure, and the Area required in another: I ſhall therefore give ſome Examples of each.

I. *For Board.*

IF the Length and Breadth be both given in Inches, and the Content ſought in Feet. *Say,*

As 144, *is to the Breadth*;
So *is the Length, to the Content.*

Example. A Board is 14 Inches broad, and 156 Inches long: How many ſquare Feet doth it contain? Anſwer 15.16. *For,*

As 144 *upon* A, *is to* 14 *the Breadth upon* B;
So is 156 *the Length upon* A, *to* 15.16 *the Content upon* B;

If the Breadth be given in Inches, and the Length in Feet: *ſay,*

As 12, *is to the Breadth*;
So *is the Length, to the Content.*

Example. A Board is 14 Inches broad, and 13 Foot long: What is the Content in Feet? Anſwer, 15.16. *For,*

As 12 *upon* B, *is to* 14 *the Breadth upon* A;
So *is* 13 *the Length upon* B, *to* 15.16 *the Content upon* A.

II. *Of*

Of Paving.

PAvier's Work is meaſured by the Yard Square, which contains 9 ſquare Feet: The moſt natural way is to meaſure the Sides with a Yard divided into 100 equal parts, and then multiply the Breadth by the Length, the Product will be the Content in ſquare Yards: But if the Sides be given in Feet, *ſay*,

As 9 *is to the Breadth*;
So *is the Length, to the Content.*

Example. There is a Court pav'd with Stone, the Breadth is 64 Foot, and the Length 95 Foot: What is the Content in ſquare Yards? Anſwer, 67.55. *For*,

As 9 *upon* A, *is to* 64 *the Breadth upon* B;
So *is* 95 *the Length upon* A, *to* 67.55 *the Content upon* B.

III. *Of Cieling.*

Cieling and alſo *Wainſcotting*, are meaſured by the Yard Square, and therefore what is ſaid in the Example above, may be underſtood here.

IV. *Of*

IV. *Of Flooring and Tyling.*

THeſe are both meaſured by the *Square*, which contains 100 ſquare Foot, the Side being 10 Foot: Therefore if the Length and Breadth be given in Feet, multiply one by the other, and cut off two Figures in the Product, the Remainder ſhews the Content in Squares, and the Figures cut off are the parts of a Square, or odd Feet.

Or by the Rule, ſay,

As 100, *is to the Breadth in Feet;*
So *is the Length in Feet, to the Content in Squares.*

Example. There is a Floor 66 Foot broad, and 75 Foot long. How many Squares doth it contain? Anſwer, 49.5. *For,*

As 100 upon A, is to 66 the Breadth upon B;
So is 75 the Length upon A, to 49.5 the Content upon B.

The like for Tyling.

Laſtly, The Breadth of any Superficies being given in Feet or Yards, to find how many in Length (of either) it will take to make a Foot or Yard ſquare.

The Proportion is,

As *the Breadth is to* 1;
So *is* 1, *to the Length ſought.*

Example.

Example. A Board is 1.5 Foot broad: How much in Length will it take to make a Foot ſquare? Anſwer, .667. *For,*

As 1.5 (*the Breadth upon*) A, *is to* 1 *upon* B;
So *is* 1 *upon* A, *to* .667 *upon* B.

So that if one ſide of a piece of Board be 1.5 Foot, the other ſide muſt be .667 of a Foot, to make a Square-Foot; for .667 multiplied by 1.5 is 1.

In Inch-meaſure the Proportion is,

As the Breadth in Inches, is to 12;
So is 12
To the Length in Inches to make a Foot.

And, *As the Breadth in Inches is to* 36,
So is 36
To the Length in Inches to make a Yard.

Example. A Board is 18 Inches broad, how many Inches in Length will make a Foot?

Set 18 (the Breadth) upon A, to 12 upon B, then againſt 12 upon A, is 8 upon B; and ſo many Inches in Length will make a Foot of this board; for 18 multiplied by 8, gives 144 the Square-Inches in a Square-Foot.

APPEN-

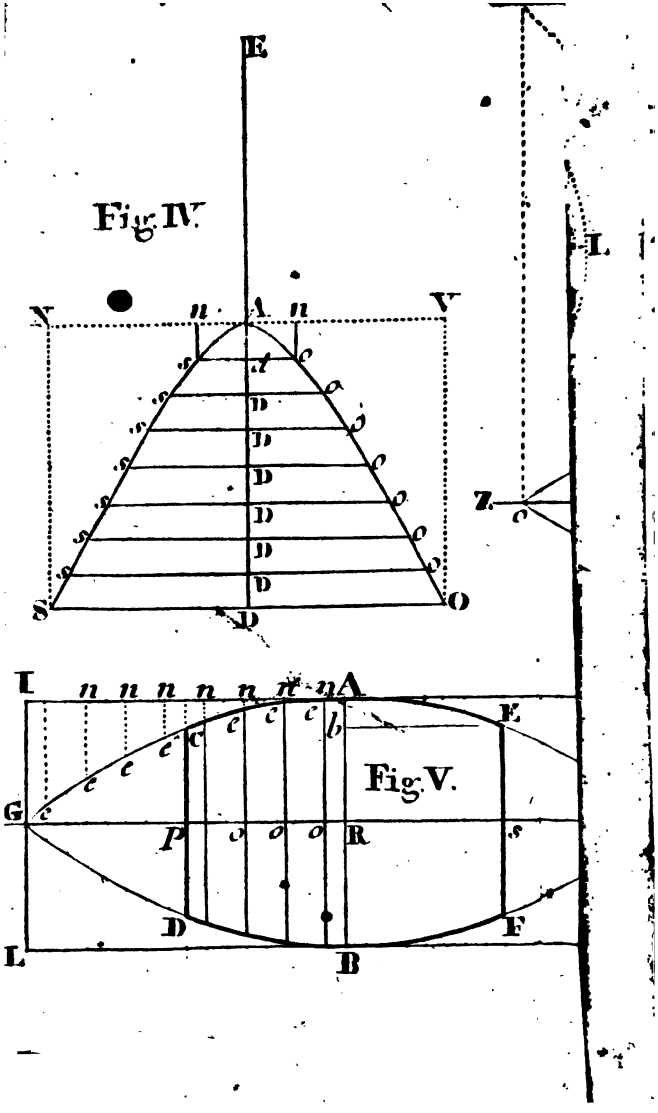

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Fig. IV.
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APPENDIX.

Of Conical Sections.

IN the Doctrine of *Conicks* there are these two things (very different) to be separately considered. *First*, What Figures or Curve-Lines do arise from the Section of a Cone by a Plain. And *Secondly*, What is the nature of such Figures or Curve-Lines, considered without respect had to such supposed Original, by cutting a Cone.

See Dr. *Wallis de Con. Sect.* and Treatise of *Algebra*, pag. 292.

DEFINITION.

1. A Cone *is a Solid described, when one of the containing Sides of a Rectangled Triangle remaining fixed, the Triangle is turned round about till it return to the place from whence it first moved: The Axis of a Cone is that fixed Side about which the Triangle is moved: The Base of the Cone is a Circle, which is described by the containing Side that is moved about.*

Of the ſeveral Sections of a Cone.

1. IF a Cone be cut by a Plain through its Axis, from the Vertex to the Center of the Baſe, the Section will be a *Triangle* as S V B. Fig. 1 and 2.

2. If a Cone be cut by a Plain at Right Angles to its Axis, (or if a Right Cone, Parallel to its Baſe) the Section will be a *Circle*, whoſe Diameter let be S d B, Fig. 1.

3. If a Cone be cut by a Plain Parallel to the ſide of the Triangle (S V) the Crooked Section C c c N c c C, will be a *Parabola*; in which P N, is the Diameter, or Axis, and d c an ordinately applied Line. *Vid.* Fig. 1.

4. If a Cone be cut by a Plain that will alſo cut one of the ſides of the Triangle produced in E, (Fig. 1.) the Crooked Section O o o A o o O, is a *Hyperbola*, in which A H is the intercepted Diameter: A E the Tranſverſe Diameter; and b o, an ordinately applied Line.

5. If a Cone be cut by a Plain through both the ſides of the Triangle under the Vertex V, the Crooked Section, a o o A o o will be an *Ellipſis*, in which a A is the Diameter, and D o, and ordinately applied Line. *Vid.* Fig. 2.

There are all the Sections that can be made by cutting a Cone by a Plain. I ſhall treat only of the four laſt, and of theſe but ſo much as I conceive neceſſary to my preſent Deſign, which is only

only to explain the Nature and Original of ſome Solids which are frequently mentioned in Books and do ſometime occur in the Practice of Gauging.

Of a Parabola.

IN the *Parabola* (Fig. 1.) Let the Cone be cut by a Plain (S d B) parallel to the Circular Baſe (S P B) it will appear,

1. That S d is equal to S P, for N P is parallel to S V, and S d B, is parallel to S P B.

2. That S P multiplied by P B is equal to P C ſquared. Therefore S d × d B = d c q.

3. *As* N P, is to P C ſquared;
So is N d, to d c ſquared. Therefore,

4. *As* N P, is to N d;
So is P C ſquared, to d c ſquared, which is the general Property of a *Parabola*.

Problem I. Figure 3.

To deſcribe a Parabola *in* Plano.

DRaw the ſtraight Line O P O, and upon the middle Point P, erect the Perpendicular P A, compleat the Triangle by drawing the Lines A O, A O, then through A P (the Axis) and parallel to O P O (the Baſe) draw a competent number (the more the better) of ſtraight

Lines S D S. Now whereſoever D be taken, it will be

A P : P O : : A D : D e. And
P O × D e = D o q.

This gives the Points o. o. (as many as you pleaſe on both ſides) through which the Curve Line O o o A o o O being drawn, you have the *Parabola* required.

Or thus,

Having drawn the *Parallelogram* O N V O, (Fig. 3.) divide A N into any number of equal parts, and ſuppoſe A P divided into as many equal parts as the Square of the number of parts in A N, as here A N is divided into 4 equal parts, and A P is ſuppoſed to be divided into 16 equal Parts. Let Lines be drawn from N. n. n. n. parallel to A P. Now, if from a Scale of equal Parts, of which 16 are equal to A P, you make the firſt n c = 1. the ſecond n c = 4. the third n c = 9. and the fourth N O = 16. the Points C. C. C. O. are in the *Parabola*, and by a competent number of Points thus found, the *Parabola* may be drawn.

Note, 1. The *Parabola* O c A O, is two thirds of O N V O, the *Parallelogram* circumſcribed, and the Triangle O A O, is three fourths of the *Parabola*; therefore if the *Parallelogram* be 3, the *Parabola* is 2, and the *Triangle* 1.

2. If the *Parallelogram* P A N O, be turned about the Line A P, it will generate the *Cylinder*

 O N V O,

ONVO, and by the ſame converſion there will be generated by the Curve-Line O c c c. the *Parabolick Conoid*, O C A O O, and by the ſtreight Line O A, the Cone O A O, and theſe three Solids are as 3. 1 and a half, and 1; for the *Parabolick Conoid*, is half the circumſcribing *Cylinder*, and the *Cone*, one third part thereof. Now, all theſe properties being common to every *Parabola* (as is demonſtrated by Dr. *Wallis* in his *Mechanica*, Part 2 Chap. 5.) the Content of any *Parabolick Conoid* is eaſily found, as ſhall be ſhewn in the next Problem.

Problem II.

There is a Parabolick Conoid (Fig. 3.) O A O, *the Diameter of the Baſe is* 32 *Inches, and* P A, *the Altitude* 45.7143 *Inches; What is the Content in Ale-Gallons?*

MUltiply the Area of the Baſe by half the Altitude, the Product gives the Content ſought.

Example. The Diameter is 32. The Area is 2.8519, this multiplied by half the Altitude (*viz.*) 22.8571 gives 65.18 the Content ſought.

Or, Multiply the Square of the Diameter by half the Altitude, and that Product by .0027851, the laſt Product will be the Content in Gallons.

Problem III.

There is a Fruſtum of a Parabolick Conoid, (Fig. 3.) *whoſe Baſes are Parallel*; O P O, *the Diameter of the greater Baſe is* 32 *Inches*: C D o, *the Diameter of the leſſer Baſe* 24 *Inches, and* P D *the Altitude* 20 *Inches*: *What is the Content in Ale-Gallons?*

FRom the ſquare of P O (the Semidiameter of the greater Baſe) ſubtract the ſquare D O (the Semidiameter of the leſſer Baſe) then ſay,

As that Difference,
Is to P D the Altitude of the Fruſtum;
So is the Square of P O,
To P A the Axis of the whole Conoid.

In this Example it will be

P o q. — D o q: P D:: P o q: P A.
112 : 20 :: 256 : 45.7143

Now, if from P A you ſubtract P D, you have the Altitude of the *Parabolick Conoid*, whoſe Baſe is C D O. And ſince every *Parabolick Conoid* is equal to half a *Cylinder* of the ſame Baſe and Altitude, the Area of the Baſe O P O multiplied by half the Altitude P A, gives the Content of the *Parabolick Conoid*, whoſe Baſe is O P O, and Altitude A P. And the Area of the Baſe C D O, multiplied by half the

the Altitude D A, gives the Content of the *Parabolical Conoid* whose Base is C D o, and Altitude D A. Therefore, if from the Content of the former you subtract the Content of the latter, you have the Content of the Frustum C D o, O P o.

Example. C D o is 24 Inches, the Area is 1.6042, this multiplied by half D A (*viz.*) 12.8571, gives 20.62 Gallons, the Content of the *Parabolick Conoid* D C C A O O, this subtracted from 65.18 (the Content of the *Parabolick Conoid* O C A o o) leaves 44.56 Gallons, the Content of the Frustum (c D o. O P O,) which was demanded. But the Content of any such Frustum may be more readily found, thus,

To the Area of the greater Base, add the Area of the lesser Base, the Sum multiplied by half the Altitude, gives the Content.

Example. In the Frustum last mentioned.

The Diameter O P O, being 32 Inches, the Area is	2.8579
The Diameter C D o, being 24 Inches, the Area is	1.6042
Sum is	4.4561

This multiplied by half the Altitude (*viz.* 10.) gives 44.561 Gallons the Content, as by the last.

Now, Suppoſe the Fruſtum O B G O, be equal to the Fruſtum O C D o O, and the Baſe O P O, common to both, this is what ſeveral Writers would have to repreſent a common Cask, *viz.* a *Butt*, *Hogſhead*, *Barrel*, *&c.* But it is plain from the nature and conſtruction of the Figure, that no ſuch Cask can be in the Form of theſe two Fruſtums abutting upon the common Baſe O P O. But if it were practicable to make a Cask in this Form, if the Diameters, and the Length be as above-mentioned, the Content will be 89.122 Gallons.

To find the Content upon every Inch of the Fruſtum. C D o. O P O.

From the Square of (OPO) the Diameter of the greater Baſe, ſubtract the Square of (CDo) the Diameter of the leſſer Baſe, and divide this difference by D P the Depth, the Quotient is the common Difference of the Squares of the ſeveral Diameters, taken one in the middle of every Inch of D P. But for the Diameter in the middle of the firſt Inch next the leſſer Baſe, take half this common Difference, and add it to the Square of CDO, the Sum is the Square of the Diameter in the middle of that Inch, to this add the common Difference, the Sum will be the Square of the Diameter in the middle of the ſecond Inch, and ſo of the reſt. But having got the Squares of the Diameters in the middle of the firſt, ſecond, and third Inch, find the Area's of thoſe Circles in Gallons, the Difference of theſe Areas will be equal.

Example.

Example.

O P O = 32 Square =	1024
C D O = 24½ Square =	576
Difference =	448

448 divided by D P 20, is 22.4 the common Difference.

Square of C D O =	576
half com. differ. =	11.2
Square of the Diameter in the middle of the first Inch =	587.2
Common Difference =	22.4
Square of the Diameter in the middle of the second Inch =	609.6
Common Difference =	22.4
Square of the third Diameter =	632.0

The Squares of these Diameters thus found, multiplied severally by .002785, give the Area or Content of the first, second, and third Inch.

		Areas.	Diff.
Viz.	1	1.6354	
	2	1.6978	.0624
	3	1.7602	.0624

And so of the rest, as in the following Table; in the last Column of which are the common Difference of the Areas. In the third Column, you have the Area or Content of every Inch. The

first Number in the second Column, *viz.* 44.5640 is the Sum of all the Area's in the third Column, that is, the Content of the whole Tun: From which the several Area's continually subtracted, leaves the number of Gallons contained in the Tun at any Depth. Thus against 10 Inches deep, you have 25.402 Gall. in the Tun, and so for any other Depth.

Inches of the Tun's depth.	Content at any depth.	Area's.	Com. Diffs.
20	44.5640	1.6354	.0624
19	42.9286	1.6978	.0624
18	41.2308	1.7602	.0624
17	39.4706	1.8226	.0624
16	37.6480	1.8850	.0624
15	35.7630	1.9474	.0624
14	33.8156	2.0098	.0624
13	31.8058	2.0722	.0624
12	29.7336	2.1346	.0624
11	27.5990	2.1970	.0624
10	25.4020	2.2594	.0624
9	23.1426	2.3218	.0624
8	20.8208	2.3842	.0624
7	18.4366	2.4466	.0624
6	15.9900	2.5090	.0624
5	13.4810	2.5714	.0624
4	10.9096	2.6338	.0624
3	8.2758	2.6962	.0624
2	5.5796	2.7586	.0624
1	2.8210	2.8210	.0624
Content = 44.5640			

Of

Of a Parabolick Spindle.

IN Fig. 5. R H is 40, R S = b E 20. A R 16, R b 12, and b H 4. And

A R : R Hq : : A h : b Eq.
16 : 1600 : : 4 : 400.

Therefore (the other ſide being made like to this) the Curve Line GCAEH is a *Parabola* Now if this *Parabola* be turned about upon the Line GH, it will generate the Solid GAHBG, which is called a *Parabolick Spindle*. And every *Parabolick Spindle* is to its circumſcribing Cylinder, as 8 to 15. (*Vid.* Dr. *Wallis's Mechanica*, Part II. Chap. 5. Prob. 12.)

Problem IV. Figure 5.

To deſcribe a Parabolick Spindle, *the middle Fruſtum whereof ſhall Repreſent a Cask whoſe Diameter at the Bung* A B, *is* 32 *Inches, the Diameters at the Heads* C D *and* EF *are each* 24 *Inches, and the Length* P S 40 *Inches. And to find the Content of the middle Fruſtum thereof, repreſenting the Cask propoſed.*

1. DRAW the ſtraight Line z z, and at right Angles to it, draw the Lines A B, C D, and E F, ſo as R P and R S, be each = 20. Make

Make A R = 16. C P, b R and E s, each = 12, ſo ſhall A b be = 4. And becauſe it is a *Parabola*, it will be

A b : A R : : b E q : R H q
4 : 16 :: 400 : 1600

The Square-Root of 1600, is 40, = R H or R G.

2. To find a Diameter in the middle of every Inch of R G, or R H. Through the Point A, draw I K Parallel, and equal to the Line H G, let I A and G R, be each divided into half Inches, which here will be 80, the Square of this is 6400, by which divide A R 16, the Quotient will be .0025, this multiplied by the Squares of all the odd Numbers, *viz.* 1, 3, 5, 7, &c. to 79, the ſeveral Products give the Length of the Lines *n e*, which taken from the Line A R, leaves the Lines *e o*, the Semidiameters of the *Parabolick Spindle*, (one in the middle of every Inch of G R.) But to contract this Work, we need but the three firſt Diameters, the reſt are obtained by ſubtracting the common Difference; as will appear by the following Operation.

Multiplied by { 1, 9, 25 } is { .0025, .0225, .0625 } each of

theſe taken from A R 16, the three Remainders will

be { 15.99975, 15.9775, 15.9375 } theſe multiplied by 2 give { 31.9995, 31.955, 31.875 } the

three

three Diameters, one in the middle of every Inch next to A B; to find the common Difference, take the ſecond Diameter from the firſt, there reſts .040; take the third from the ſecond, the Remainder is .080, take .040 from .080. there reſts .040, the ſecond Difference, which is the ſame throughout; and by this common Difference the reſt of the Diameters are thus found. Take the firſt Difference (*viz.* .040) from the firſt Diameter, the Remainder is the ſecond Diameter; add the ſecond Difference to the firſt Difference, and ſubtract their Sum from the ſecond Diameter, the Remainder will be the third Diameter; and ſo of the reſt, as in the following Table. In the ſecond Column of which you have the ſecond Difference (which is the ſame throughout) in the third Column are the firſt Differences, in the fourth Column you have the true Diameters of the Spindle, one in the middle of every Inch of R P. and in the laſt Column are the Areas to theſe Diameters, that is, the Content of ſo many circular Plains of one Inch thick, which added together make 48.249 Ale-Gallons, the Content of the *Part* B D C A, this doubled is 96.498. the Content of the middle Fruſtum C A E F B D, repreſenting the Caſk propoſed.

Note, The Areas in this Table were found by multiplying the Squares of each Diameter by .002785. But this Work may be much ſhorten'd; for if the Squares of the ſeveral Diameters in the fourth Column be added together, and their Sum multiplied by .002785, the Product will be 48.349 = the Sum of the Areas in the laſt Column.

A Table shewing the Content of half the middle Frustum of the *Parabolick Spindle*, CAEFBD. Fig. 5.

	Second Differ.	Firſt Difference.	Diameter in the middle of every Inch of R P.	Area in Gallons, or the Content of every Inch from R to P.
1			31.995	2.851
2	40	40	31.955	2.844
3	40	80	31.875	2.830
4	40	120	31.755	2.808
5	40	160	31.595	2.780
6	40	200	31.395	2.745
7	40	240	31.155	2.703
8	40	280	30.875	2.655
9	40	320	30.555	2.600
10	40	360	30.195	2.539
11	40	400	29.795	2.473
12	40	440	29.355	2.400
13	40	480	28.875	2.322
14	40	520	28.355	2.239
15	40	560	27.795	2.152
16	40	600	27.195	2.060
17	40	640	26.555	1.964
18	40	680	25.875	1.865
19	40	720	25.155	1.762
20	40	760	24.395	1.657
			Sum	48.249
			Content of the Cask	96.498

Note, If the part A B D C were a Tun ſtanding upon its leſſer Baſe D C, and from the whole Content thereof, *viz.* 48.249, you ſubtract the firſt Area, *viz.* 2.851, the Remainder 45.398, will be the Content at 19 Inches deep; and if from this you take the ſecond Area, you will have 42.554, the Content at 18 Inches deep, and proceeding with the reſt in the ſame manner, you will have a Table ſhewing the Content at any Depth.

Of an Hyperbola.

IN Fig. 1. the *Cone* S V B, is cut by a Plain H A, which alſo cuts one Side of the Triangle produced in E, therefore the crooked Section O o o A o o O, is an *Hyperbola*. In which E A is the tranſverſe Diameter, and A H the intercepted Diameter. Now if the *Cone* be cut by another plain Parallel to the Baſe, this Section will be a Circle, whoſe Diameter is S H B: And S H × H B = H O q, therefore H O is an ordinately applied Line, and if H O be a Line ordinately applied, it will be

A H × E H : A b × E b :: H O q : b o q.

Problem

Problem V. Fig. 4.

To find the Content of an Hyperbolick Conoid, *or the Content of any part thereof in Ale-Gallons.*

IF the *Semi-Hyperbola* A S S S D, be turned round about upon the Line A D, it will deſcribe the *Hyperbolick Conoid*, S S A o O. Now, if we put the Tranſverſe Diameter = t, the Intercepted Diameter = d, the Magnitude of the *Hyperbolick Conoid* = A, and the Magnitude of a Cylinder of the ſame Baſe and Altitude = B, it will be

$$6t + 6d : 3t + 2d :: B : A.$$

That is, (1.) To 6 times the Tranſverſe Diameter A E, add 6 times the intercepted Diameter A D, the Sum is your Diviſor.

(3.) To 3 times the Tranſverſe Diameter, add twice the Intercepted Diameter, by this Sum multiply the Content of the Cylinder N V O S, the Product is your Dividend; which divided by the Diviſor above found, the Quotient will be the Content of the *Hyperbolick Conoid*, whoſe Baſe S D O, is the Baſe of the Cylinder N V O S, and Altitude A D equal to S N the Altitude of the Cylinder; and this will hold whereſoever the Baſe be taken: For ſuppoſe the firſt S d o. next the Vertex A, be taken for the Baſe of the *Hyperbolick Conoid*, S A o, (and of the Cylinder S n n o,) the Content of this *Hyperbolick Conoid* may be found as above, and ſo the Content of any Fruſtum, S D O o D S may be likewiſe found,

found, for if the little *Hyperbolick Conoid* S A o d, be taken from the *Hyperbolick Conoid* S S S w o o O, there remains the Fruſtum S D O o d s, and thus the Content of the whole, and alſo the Content of any Fruſtum, and conſequently of every Inch of the *Hyperbolick Conoid* may be readily found, as will appear in the following Example.

Example. Fig. IV.

Let A E, the Tranſverſe Diameter, and A D, the Intercepted Diameter, be each 32 Inches, and S D O the Diameter of the Baſe 45.255 Inches, to find the Content of the *Hyperbolick Conoid* in Ale-Gallons.

1. Six times A E is 192, and 6 times A D is 192, the Sum of theſe is 384, for the firſt Term of the Proportion.

2. Three times A E is 96, and twice A D is 64, the Sum of theſe is 160, for the ſecond Term.

3. The Diameter of the Baſe S D O is 45.255 (equal the Square-Root of 2048) and the Area of this Circle is 5.704 Gallons; this multiplied by 32 (equal the Altitude S N) gives 182.528 the Content of the Cylinder NSOV, and this is the third Term of the Proportion, which will ſtand thus;

384 : 160 : : 182 : 528.

Now multiply the *third* Term by the *ſecond*, and divide that Product by the *firſt*, the Quotient or *fourth* Term will be 76.053 Ale-Gallons, which is the Content of the *Hyperbolick Conoid* required.

And by the ſame Method the Content of the little *Hyperbolick Conoid*, whoſe Altitude d A, is 4 Inches, and the Diameter of its Baſe (S d o) 16.97 Inches,

Inches, will be found to be 1.545 Gallons. This taken from the Content of the *Hyperbolick Conoid* above-found, the Remainder is 74.508 Ale-Gallons, the Content of the Fruſtum S D O o d S; and ſo for any other Fruſtum.

Problem VI. Figure 6.

To deſcribe an Hyperbola *in* Plano *Geometrically.*

DRaw a ſtraight Line R R, and from the middle Point W, let fall the Perpendicular W B, then divide W R in any number of Parts, and draw the Lines r o G, r o H, parallel to W B, then ſet one Point of your Compaſſes in the Line W B, ſuppoſe at A, and extend the other to r, where keep it faſt whilſt the firſt Point, with the ſame Extent, is removed from A, to the Line r G, it will fall upon the Point O, and ſo on the other ſide. I ſay, the Points o o o, are in the *Hyperbola*, and by a competent Number of ſuch Points thus found, the *Hyperbola* may be drawn. W A is half the Tranſverſe Diameter, A B the intercepted Diameter, and B o an ordinately applied Line, *(Vid.* Dr. *Wallis*'s *Mechanica*, Part II. Pag. 553.)

Note, If the *Hyperbola* O o A o O, be turned about upon the Line z z, it will deſcribe a Solid O o A o o O F D E Q, which may be called a *Hyperbolick Spindle.*

Problem

Problem VII. Fig. 6.

To describe a Hyperbolick Spindle, *the middle Fruſtum of which ſhall repreſent a Cask, whoſe Diameter at the Bung* A D, *is* 22 *Inches, the Diameters at the Heads,* O E, *and* O F *are each* 24 *Inches, and the Length* G H, 40 *Inches, and to find the Content of this Fruſtum in Ale-Gallons.*

DRaw the right Line z z, and at Right Angles thereto, the Line W D, make B G and B H each equal to 20 (the Semi-length of the Cask propoſed) through the Points G and H, draw the Lines r E, and s F parallel to W D, make G C and H c, each equal to 12, and B A 16, then make B b equal to G o, ſo ſhall A b be 4. I ſay, the Square b o, (400) divided by A b, (4) is equal to 100, from which ſubtract 4, the Remainder (96) is the *Tranſverſe Diameter* of this *Hyperbola*, make A W equal to 48, (the half of 96) and through the Point W, draw the Line R R parallel to z z, and let the Line r o be parallel to W D; I ſay, the Square of W A, more the Square of W r, is equal to the Square of r o, of which the Square-Root is the Length of the Line r o. Now the Line r o, taken from the Line W B, the Remainder is the Line o g, which doubled gives o e, a Diameter in the *Spindle*; and this will hold whereſoever r be taken, and ſo innumerable Diameters o e, may be found on both Sides A D, to the ends of the Spindle. But for our preſent purpoſe, it will be ſufficient to find

find a Diameter in the middle of every Inch of the part A D E O, as ſuppoſe the firſt Diameter o g e, be half an Inch from A D, the firſt W r is .5, the Square of this (*viz.* 25.) added to the Square of W A (2304) is equal to the Square of r o, (2304.25) the ſecond W r, will be 1.5, the third 2.5, *&c.*) the Squares of which you have in the firſt Column of the following Table. In the ſecond Column are the Sums of the Squares of W A and W r, that is, the Squares of r o. In the third Column are the Square-Roots of the Numbers in the ſecond Column, that is, the Length of the Lines r o. Now each of theſe Numbers taken from W B 64, leave the Lines o g, which doubled are the Diameters o e, as in the fourth Column. In the laſt Column are the Areas of theſe Circles in Ale-Gallons, that is, the Content of ſo many Circular Plains of one Inch thick, the Sum of theſe is 49.1 (equal the part A D E O) which doubled is 96.2, equal to the middle Fruſtum O E D F O A. The Content here found is not Mathematically True, but near enough for any ordinary Practice, and by the Method here laid down, the Content may be found ſo near the Truth, that the difference ſhall be leſs than any quantity aſſignable.

Now, Let the part D E O A, repreſent a Tun, and ſuppoſe it to ſtand upon the leſſer Baſe E o. If from the whole Content thereof, *viz.* 48.1, you ſubtract the firſt Area, *viz.* 2.852, the Remainder 45.249, will be the Content at 19 Inches deep; and if from this you ſubtract the ſecond Area, *viz.* 2.843, you will have 42.406, the Content at 18 Inches deep; and proceeding with the reſt after the ſame manner, you will have a Table ſhewing the Content of this Tun at any Depth.

A TABLE shewing the Content of every Inch of half the middle Fruſtum of the Hyperbolick Spindle Eo Ao F.D (Fig. VII.) *in Ale-Gallons.*

Squares of W r.	Sum of the Squares of W A. and W r.	Square Roots of the Square of (r o) or the length of the Lines r o.	Diameters o e	Areas of the circles in Ale-Gallons.
.25	2304.25	48.003	31.995	2.851
2.25	2306.25	48.023	31.953	2.843
6.25	2310.25	48.065	31.870	2.829
12.25	2316.25	48.127	31.745	2.807
20.25	2324.25	48.210	31.579	2.777
30.25	2334.25	48.314	31.372	2.741
42.25	2346.25	48.438	31.124	2.698
56.25	2360.25	48.582	30.835	2.648
72.25	2376.25	48.747	30.506	2.592
90.25	2394.25	48.931	30.138	2.520
110.25	2414.25	49.135	29.730	2.462
132.25	2436.25	49.358	29.283	2.388
156.25	2460.25	49.601	28.798	2.310
182.25	2486.25	49.862	28.275	2.227
210.25	2514.25	50.142	27.715	2.139
240.25	2544.25	50.441	27.119	2.048
272.25	2576.25	50.757	26.486	1.954
306.25	2610.25	51.090	25.819	1.857
342.25	2646.25	51.442	25.117	1.753
380.25	2684.25	51.809	24.380	1.656
Sum is				48.100
This doubled is				96.200
The Content of the Fruſtum Eo Ao E D.				

Of

Of an Ellipſis.

IF the *Cone* S e B d V (Fig. 2.) be cut by a Plain (A a) through both the Sides of the Triangle, the crooked Section a o o A o o will be an *Ellipſis*, in which A a, is the Tranſverſe Diameter, or Axis, and A D, a Diameter intercepted.

Let the *Cone* be cut by another Plain (S B) parallel to the Baſe S e B d, it will be S D x B D = D o q, and this will hold whereſoever D be taken, and give the Lines D o, which are called Ordinately applied Lines, and the Lines o D o (cutting the Line a A, at Right Angles) are called Conjugate Diameters of the *Ellipſis*.

Problem VII. Fig. 7.

Given the Axis of the Ellipſis, *and its greateſt Conjugate Diameter* (*cutting the Axis at Right Angles*) *to deſcribe the* Ellipſis.

DRAW a ſtraight Line, and make A C, equal to the Axis, given, through the middle Point B, and at Right Angles to A C, draw the Line F L, and from the Point B, ſet off half the Conjugate Diameter to O and G, draw the Lines D H, d h, parallel to F L. Join B D, and B d, upon the Center B, and with the interval B A, deſcribe the Circle F A L C. I ſay, B D q — B E q = E D q. And B F q : B O q : : E D : E O. This will hold whereſoever E be taken, and give the Points

Points o o, (as many as you pleaſe on both Sides) through which the *Ellipſis* may be drawn.

Of a Spheroid. Fig. VII.

IF the *Semi-Ellipſis* A O C, be turn'd about upon the Line A C, it will generate a Solid A G C O, which is call'd a *Spheroid.*

Note, 1. The Content of every *Spheroid* is equal to two thirds of the Content of a Cylinder, whoſe Baſe is equal to the greateſt Circle of the *Spheroid*; and its Altitude equal to the *Spheroid*'s Axis. Therefore the *Area* of the greateſt Circle of a *Spheroid*, multiplied by two thirds of its *Axis* gives the Content.

2. If the *Spheroid*, be cut by a Plain Z Z, parallel to the Line O G, and through the middle of B C, the Content of the Fruſtum Z C Z, to the Content of a Cylinder, whoſe Baſe is Z Z, and Altitude V C, is as 5 to 9. Therefore, the Area of the Baſe Z Z, multiplied by five Ninths of V C, gives the Content of the Fruſtum Z C Z.

3. If the *Spheroid* be cut as above, the upper Fruſtum ZOAHGZ, is equal to a Cylinder, whoſe Baſe is Z Z, and its Altitude V A. Therefore the Area of the Baſe Z Z, multiplied by the Altitude V A, gives the Content of the Fruſtum Z O A G Z. Theſe three Propoſitions do alſo hold in a *Sphere*, *Vid.* Dr. *Wallis*'s *Treatiſe of Algebra*, *Page* 312.

Problem

Problem VIII. Fig. 7.

To deſcribe a Spheroid, *the middle Fruſtum of which ſhall repreſent a Cask, whoſe Bung Diameter* O G, *is* 32 *Inches, the Diameters, at the Heads* O H *and* O K, *are each* 24 *Inches, and the Length* E E 40 *Inches. And to find the Content of ſuch Cask upon every Inch of the Depth* E E.

FRom the Square of (O G) the Bung-Diameter, Subtract the Square (O H) the Head-Diameter. Then,

As that Difference,
Is to the Square of O G,
So is the Square of B E,
To the Square of B A.

The Square-Root of B A, is the Semi-Axis of the *Spheroid*, with which, as Radius, deſcribe the Circle F A L C, then find the Points o o, as in the laſt *Problem*, and ſo the Diameters O H. O h, as many as you pleaſe, one in the middle of every Inch of E B, will be ſufficient: For the Area's of theſe Circles in Gallons, being added together, gives the Content of the Part O G H o, and this doubled is the Content of the Cask O H K O.

But for our preſent purpoſe it will be ſufficient to find the Square of a Diameter in the middle of each of the three firſt Inches, from B towards K, the Squares of the reſt, and alſo the Areas of their

their Circles being eaſily obtained by their Common Difference. Thus,

From the Square of (OG) the Bung Diameter, ſubtract the Square of (OH) the Head Diameter, and divide the Square-Root of the Difference by (BE) the Semi-Length of the Cask. Then multiply this Quotient by .5, 1.5, and 2.5, and call the Products S.

Now, if from the Square of (OG) the Bung-Diameter, you ſubtract the Squares of S, the Remainders will be the Squares of the reſpective Diameters ſought.

OG = 32	Square is =	1024
OR = 24	Square is =	576
	Difference =	448

The Square Root of 448, is 21.16601, this divided by (BE) 20, gives 1.0583, which multiplied

by .5	is	.52915	= to S.		.28
1.5		1.58745	which		2.52
2.5		2.64575	Squared is		7.00

Now the Square of OG, *viz.* 1024,

Leſs the Square of S. *viz.*	.28	is	1023.72
	2.52		1021.48
	7.00		1017.00

which are the Squares of the three firſt Diameters, one in the middle of every Inch from OG, towards O K, or OH; and theſe multiplied ſeve-

verally by .0027851, gives the Area's of the reſpective Circles in Ale-Gallons.

$$Viz. \left\{\begin{matrix} 2.85116 \\ 2.84493 \\ 2.83244 \end{matrix}\right\} \begin{matrix} .00624 \\ .01248 \end{matrix} \quad .00624$$

Now if the ſecond Inch be ſubtracted from the firſt, the Difference is .00624, and the third Inch ſubtracted from the ſecond, the Difference is .01248. then, from .01248, ſubtract .00624, the difference will be .00624, which is the ſame throughout, as in the following Table. In which obſerve, that the firſt Differences are made by a continual Addition of the ſecond Difference, and the Area's in the ſecond Column are obtain'd by ſubtracting the firſt of the firſt Differences from the firſt Area, and ſo on. Theſe Area's added together are 48.724 Gallons, equal the Content of the part O G K O, which doubled is 97.448 the Content of the Cask OHKO, which was required.

A

A Table shewing the Content of every Inch of half the middle Fruſtum of the *Spheroid*, OOKG, Fig. VII.

	Content of every Inch of the Part OGK O	1. Differ.	2. Differ.
1	2.85116		
2	2.84492	.00624	.00624
3	2.83244	.01248	.00624
4	2.81372	.01872	.00624
5	2.78876	.02496	.00624
6	2.75756	.03120	.00624
7	2.72012	.03744	.00624
8	2.67644	.04368	.00624
9	2.62652	.04992	.00624
10	2.57036	.05616	.00624
11	2.50796	.06240	.00624
12	2.43932	.06864	.00624
13	2.36444	.07488	.00624
14	2.28332	.08112	.00624
15	2.19596	.08736	.00624
16	2.10236	.09360	.00624
17	2.00252	.09984	.00624
18	1.89644	1.0608	.00624
19	1.78412	1.1232	.00624
20	1.66556	1.1856	

Total = 48.724

The Double is 97.448 = Content of the Cask.

Of a Circular Spindle.

IF the Arch of a Circle Z A Z (Fig. 8.) be turn'd about upon its Cord Z B Z, it will deſcribe a Solid Z O A O, Z F C E, and this Solid, I call a *Circular Spindle.*

Problem IX. Figure 8.

To deſcribe a Circular Spindle, *the middle Fruſtums of which ſhall repreſent a Cask, whoſe Bung-Diameter* A C, *is* 32 *Inches, the Diameters at the Heads* O E, *and* O F, *are each* 24 *Inches, and the Length* G H, 40 *Inches. And to find the Content of ſuch Cask in Ale-Gallons.*

DIvide the Square of the Semi-Length of the Cask, by half the Difference between the Bung and Head Diameters, and to the Quotient add the ſaid half-Difference; the half of this Sum is equal to the Radius A D; with which (upon D as a Center) deſcribe the Arch L A M, draw the Line L M, and make A D perpendicular thereto. Make A B equal to 16 the Semi-Diameter at the Bung. Draw the Line Z Z parallel to L M, and make B C, equal to B A. Through the Points Z C Z, deſcribe the Arch Z E C F Z, ſet off the Semi-Length of the Cask from B, to G and H, through which Points, draw Lines parallel.

llel to A D. So ſhall O E and O F, be the Head iameters of the Cask, and each 24.

To find a Diameter in the middle of every Inch om A C, towards O E. Put S = bo. It is aniſeſt, that the Square of O D, leſs the Square of is equal to the Square of O g. And if from the uare-Root of O g, you take G g, the Remainder o, will be the Semi-Diameter; which double ves the Diameter O E: : And if we put the firſt = .5, the ſecond = 1.5 the third = 2.5, &c. id proceed as here directed, we ſhall have a Dia-eter in the middle of every Inch, as in the ſecond olumn of the following Table, againſt which, in ie third Column, are the Area's in Ale-Gallons, iat is, the Content of ſo many Circular Plains of ie Inch thick, the Sum of theſe is 48.3938, the ontent of the part C E O A; which doubled is 5.7876, the Content of the Cask required.

A Table ſhewing the Content of half the middle Fruſtum of the *Circular Spindle* OECFOA. Fig. VIII.

	Diam. in the middle of every Inch of B. G.	Areas in Ale-Gall.
1	31.9952	2.8510
2	31.9567	2.8442
3	31.8797	2.8305
4	31.7642	2.8090
5	31.6098	2.7828
6	31.4166	2.7491
7	31.1843	2.7084
8	30.9126	2.6614
9	30.6012	2.6309
10	30.2497	2.5464
11	29.8577	2.4828
12	29.4248	2.4113
13	28.9505	2.3342
14	28.4340	2.2517
15	27.8749	2.1640
16	27.2723	2.0615
17	26.6255	1.9744
18	25.9336	1.8733
19	25.1947	1.7680
20	24.4016	1.6589

Sum = 48.3938

Doubled is = 96.7876 = Content of the Cask.

From

From what hath been ſaid, it appears, that the middle Fruſtum of each of the four Solids laſt mention'd, may repreſent a cloſe Cask, (*viz.* a *Butt*, *Pipe*, *Hogſhead*, &c.) And if a Cask whoſe Diameters and Length are as above ſuppoſed,

Viz. { Bung, Head } Diameter { 32, 24 } Inches.
Length ——— 40

	Ale-Gall.
I ſay, if ſuch Cask be the middle Fruſtum of a *Spheroid* (as OHKO Fig. VII.) the Content (by Prob. VIII.) will be	97.448
If the middle Fruſtum of a *Circular Spindle*, (as OEFO, Fig. VIII.) the Content (by Probl. IX.) will be	96.787
If the middle Fruſtum of a *Parabolick Spindle*, (as DCEF. Fig. V.) the Content by Probl. IV. will be	96.498
If the middle Fruſtum of a *Hyperbolick Spindle* (as OEFO, Fig. VI.) the Content by Problem VI. will be	95.200
And if (as ſome Writers ſuppoſe) the ſaid Cask be the middle Fruſtum of two *Parabolick Conoids*, abutting upon one common Baſe, (as ODOOBGO. Fig. III.) the Content by Probl. III. will be	89.122
If the middle Fruſtum of two *Cones* abutting upon one common Baſe, the Content will be	87.934

But neither of the two laſt do often (if at all) occur in the Practice of Gauging.

And though in ſeveral Books on this Subject we meet with the Terms *Fruſtum*, of a *Spheroid*, *Conoid*, *Parabolick Conoid*, *Parabolick Spindle*, &c. the Writers of ſuch Tracts have not (that I can find) told us what they mean by theſe Terms, nor will the Rules they have given find the Content of ſome of theſe Solids tolerably near the Truth.

For inſtance,

If the Cask above-mentioned be the middle Fruſtum of a *Parabolick Spindle*, the Content (as we have ſhewn above) is 96.498, Ale-Gallons.

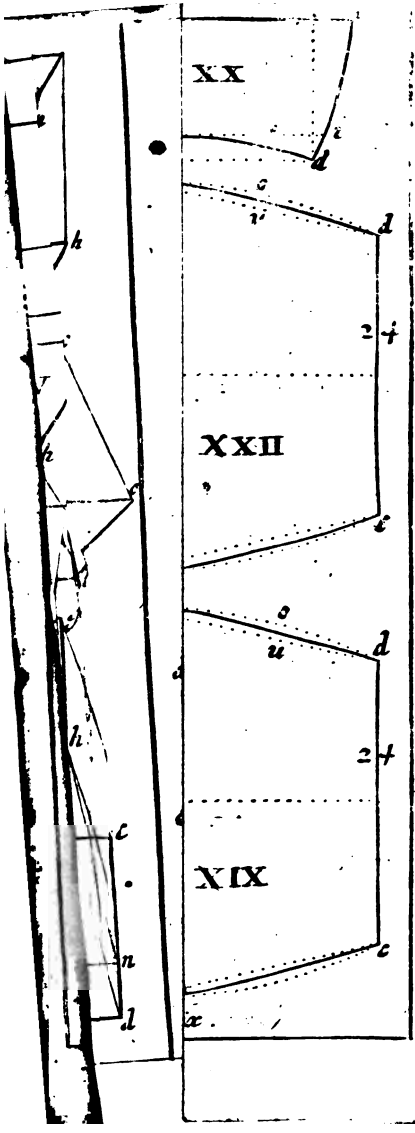
Whereas Mr. *Jordain*, in his *Duodecimal Arithmetick*, makes it but 93.877.

Mr. *Dary*, at the end of his *Miſcellanies*, makes the Content but 90.948.

And Mr. *Hunt*, in his *Practical Gauging Epitomized*, makes the Content of this Cask but 87.930, which is above 8 Gallons leſs than the true Content.

If what has been ſaid in this Appendix ſhall give any Light to the Practical Gauger, in conceiving the Nature and Properties of the Solids here mentioned, it is as much as was at firſt deſigned.

For although the middle Fruſtum of each of the four Solids deſcribed in Prob. IV, VI, VIII, and IX. do very properly repreſent a Cask, and the Contents there found be ſufficiently exact, yet the Contents of three of 'em differ ſo little one from another, that if every Cask were in the Form of one of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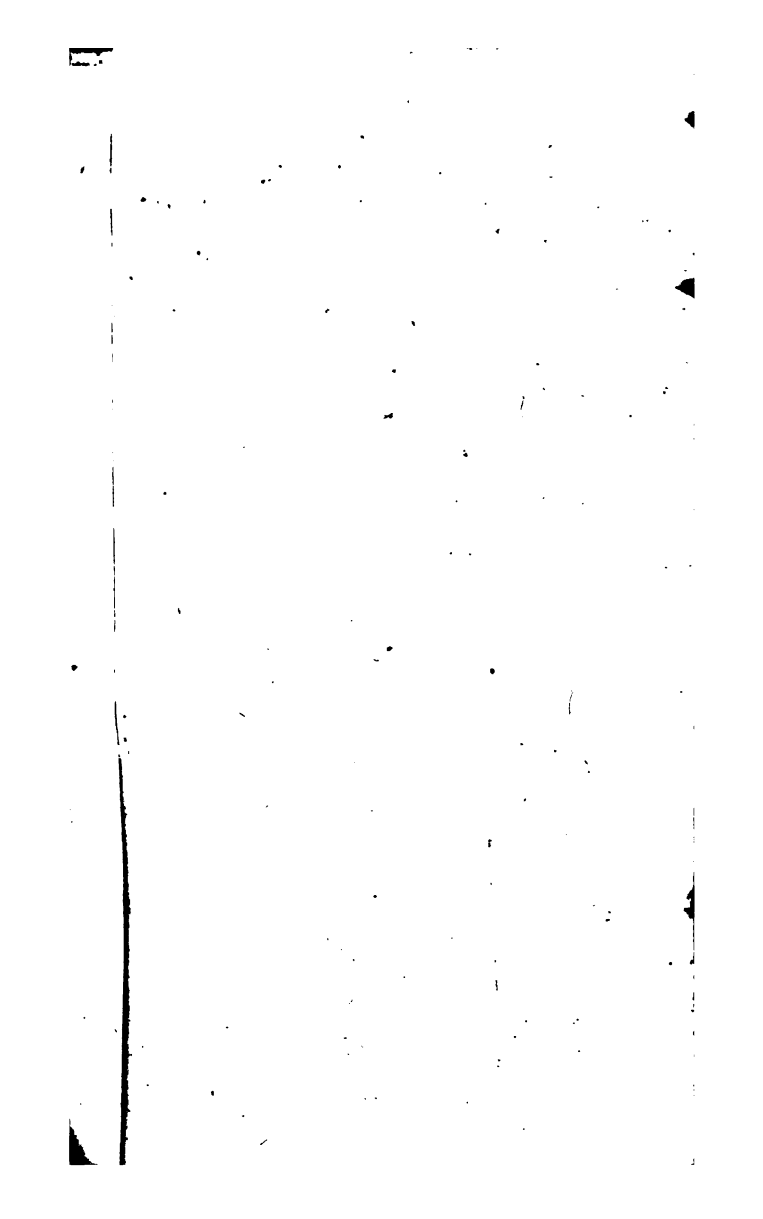

of thoſe three Solids, it would be needleſs to make any Diſtinction, becauſe upon ſight of a Cask, one could not tell to which of thoſe Solids it agreed.

And there being very few (if any) cloſe Casks of ſuch Diameters and Lengths as we have frequently mention'd, which contain more than the middle Fruſtum of a *Spheroid* (this having the greateſt Curveture between the Head and Bung) But very many Casks (of ſuch Diameters and Length) which will not contain ſo much as the middle Fruſtum of a *Parabolick* or *Hyperbolick Spindle*, (their Staves between the Head and Bung, being nearer a ſtraight Line than either of theſe Fruſtums) I muſt refer the Practical Gauger to the Rules laid down in the four firſt Problems of the Ninth Section of the foregoing Tract, by which the Content of any Cask may be readily found (either by the Pen or Inſtrument) ſufficiently near the Truth.

FINIS.